MATH 8: HANDOUT 0
INTRODUCTION AND REVIEW

WELCOME TO THE NEW SEMESTER AT SCHOOLNOVA!!

This year, we plan to study the following topics:

- Combinatorics, binomial theorem, some probability.
- Formal logic and proofs. Logic circuits.
- Euclid geometry: axioms to theorems. Triangles, quadrilaterals and circles.
- Number theory: divisibility, factorization, and modular arithmetic.
- (If time permits) some advanced topics in combinatorics (recursions and generating functions, graph theory, inclusion-exclusion).

I ask that each student bring a notebook (preferably quad ruled), pencils and a folder or binder to keep old assignments — you will need them!

As usual, all HW assignments and other information will be posted online at http://www.schoolnova.org. We will try to do much of the homework in class so that you do not need to spend too much time on it at home. **However, you should turn in complete homework, including the work done in class**.

This year we will start learning to appreciate the true rigor of Mathematics. It is enjoyable to be right, and in Math we prove a point with elegant logic instead of "winning an argument" with a dramatic opinion. If in a homework I ask you to prove a statement, please try to make it neat and tidy.

We also plan to participate in two math competitions: Math Kangaroo and American Math Contests (AMC). Math Kangaroo is an international math competition for all ages; you can find more information on their web site at http://www.mathkangaroo.org. The contest is in March. Details of the registration will follow.

AMC (https://maa.org/student-programs/amc/) is the "official" American Math Olympiad: it is the first level of the competition that eventually leads to the selection of US team for International Math Olympiad. AMC 8 is intended for students in grades 8 and below. This contest will happen in January. You do not have to register individually – just let me know if you are interested.

If you have any questions, please contact me by email: kumar@schoolnova.org.

REVIEW PROBLEMS

1. Open parentheses and expand the following expressions
   (a) $(a-b)^2 =$
   (b) $(a+b)^3 =$

2. Expand as sums of powers of x:

$$(2x+1)^2(2-3x)$$

3. Factor the following expressions:
   (a) $a^2 - b^2 =$
   (b) $a^3 - b^3 =$
   (c) $a^3 + b^3 =$

4. A coin is tossed 10 times. What is the probability of exactly 5 heads and 5 tails?

5. Consider the following quadratic equation:

$$x^2 - 7x - 8 = 0$$

   (a) What is the discriminant of this equation?
   (b) Solve the equation.

6. Write down the following fraction in a form $a + b\sqrt{5}$:

$$\frac{9 - 3\sqrt{5}}{\sqrt{5} - 2}$$

7. Which of the following numbers is the largest: $\sin 30^\circ \times \cos 30^\circ$, $\sin 45^\circ \times \cos 45^\circ$, $\sin 60^\circ \times \cos 60^\circ$?

8. If a right triangle ΔABC has sides $AB = 3\sqrt{3}$ and $BC = 9$, and side AC is the hypotenuse, find all 3 angles of the triangle.

9. Solve the equation:

$$|3x - 8| = 10$$

10. Let $x + y = 9$ and $xy = 18$
   (a) Write down the quadratic equation so that x and y are its solutions.
   (b) Calculate $x^2 + y^2$.

11. Solve the following inequality. Write your answer as a set of possible values for x.

$$\frac{(x-3)^2(x+2)}{x-4} \geq 0$$

12. A decreasing geometric series has sum 2 and the sum of its alternate terms (starting with the first) is $3/2$. What is its first term and common ratio?