Main Formulas of Combinatorics

- The number of ways to order k items is

$$k! = k(k-1)\ldots 2 \cdot 1$$

- The number of ways to choose k items out of n **if the order matters**:

$$_nP_k = n(n-1)\ldots(n-k+1) = \frac{n!}{(n-k)!}$$

- The number of ways to choose k items out of n **if the order does not matter**:

$$\binom{n}{k} = {_nC_k} = \frac{n(n-1)\ldots(n-k+1)}{k(k-1)\ldots 1} = \frac{n!}{(n-k)!k!}$$

These numbers are the ones that appear in Pascal triangle and in many other problems:

$$\binom{n}{k} = {_nC_k} = \text{The number of paths on the chessboard going } k \text{ units up and } n-k \text{ to the right}$$

$$= \text{The number of words that can be written using } k \text{ zeros and } n-k \text{ ones}$$

$$= \text{The number of ways to choose } k \text{ items out of } n \text{ if the order } \textbf{does not matter}$$

Combinatorics review problems

Note: you don't always need to compute the actual numbers, because they can be HUGE! For example, in problem 8, it's better to keep 60! ("sixty-factorial") in your solution as-is, because the actual number is larger than 10^{80}.

1. A club consisting of 20 people need to choose the president, vice-president, and treasurer. In how many ways can they do this?

2. In a meeting of 20 people, every one of them shakes hands once with every other. How many handshakes was it altogether?

3. There is a round table seating 10. How many ways there are for 10 people to choose their seats at the table? What if we do not distinguish between two seatings which only differ by rotating the table?

4. How many words one can get by permuting letters of the word "tiger"? of the word "rabbit"? of the word "common"? of the word "Mississippi"?

5. If we draw 3 cards out of the deck of 52 cards (4 suits × 13 values), what are the chances that
   - They will all be all spades
   - They will be all aces
   - That they will be ace of spades, queen of spades, and king of spades, in this order

- That they will be queen of spades, ace of spades, and king of spades, in this order
- * That they will be ace, queen, and king of spades, in some order

6. How many different paths are there on 4×4 chessboard connecting the lower left corner with the upper right corner? What about 5×5? The path should always be going to the right or up, never to the left or down.
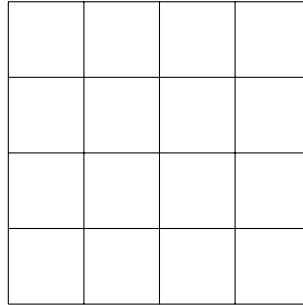
7. How many "words" of length 5 one can write using only letters U and R, namely 3 Us and 2 Rs? What if you have 5 Us and 3 Rs? [Hint: it is related to the previous problem – each such "word" can describe a path on the chessboard, U for up and R for right. . .]

8. A drunkard is walking along a road from the pub to his house, which is located 1 mile north of the pub. Every step he makes can be either to the north, taking him closer to home, or to the south, back to the pub – and it is completely random: every step with can be north of south, with equal chances. What is the probability that after 60 steps, he will end up
   (a) at the starting position
   (b) 2 steps north from the starting position
   (c) 1 steps north from the starting position
   (d) 10 steps north from the starting position
   (e) 8 steps north from the starting position

*9. You have 10 books which you want to put on 2 bookshelves. How many ways are there to do it (order on each bookshelf matters)?