MATH 7: HANDOUT 19
COORDINATE GEOMETRY 2: CIRCLES. BASIC TRANSFORMATIONS

1. Distance Between Points. Circles

The distance between two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is given by the following formula:

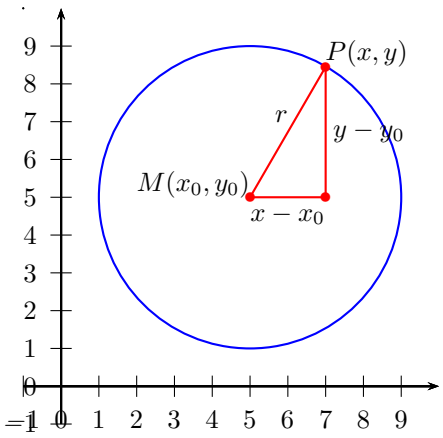
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

This formula is a straightforward consequence of the Pythagoras' Theorem.

The equation of the circle with the center $M(x_0, y_0)$ and radius r is

$$(x - x_0)^2 + (y - y_0)^2 = r^2.$$

This equation means, that points (x, y) should be at distance r from the given point $M(x_0, y_0)$.

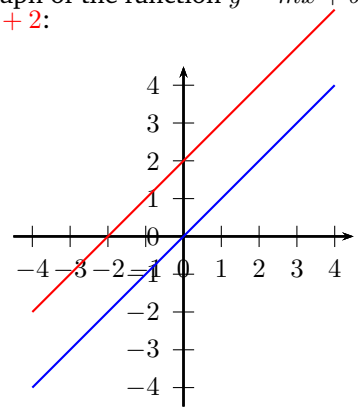


2. Graphs of functions

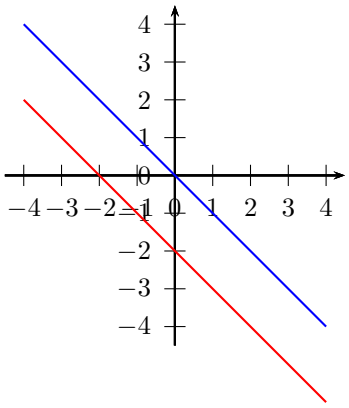
In general, the relation between x and y could be more complicated and could be given by some formula of the form $y = f(x)$, where f is some function of x (i.e., some formula which contains x). Then the set of all points whose coordinates satisfy this relation is called the **graph** of f.

Line. The graph of the function $y = mx + b$ is a straight line. The coefficient m is called the *slope*.

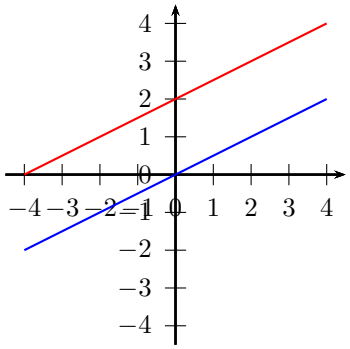
$y = x$; $y = x + 2$:



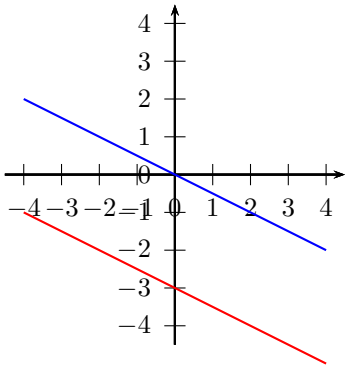
$y = -x$; $y = -x - 2$:



$y = \frac{1}{2}x$; $y = \frac{1}{2}x + 2$:

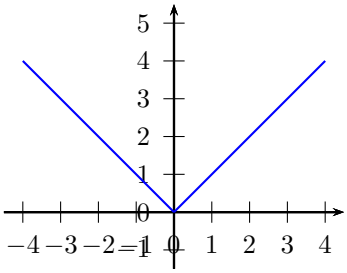


$y = -\frac{1}{2}x$; $y = -\frac{1}{2}x - 3$:



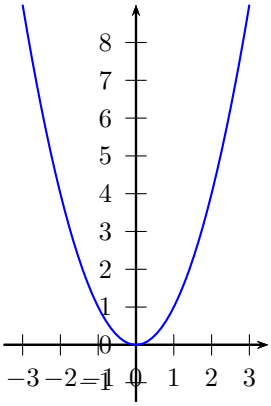
GRAPH OF $y = |x|$

The figure below shows a graph of a function $y = |x|$.



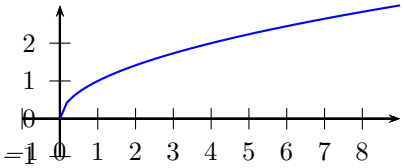
GRAPH OF $y = x^2$

The figure below shows a graph of a function $y = x^2$.



GRAPH OF $y = \sqrt{x}$

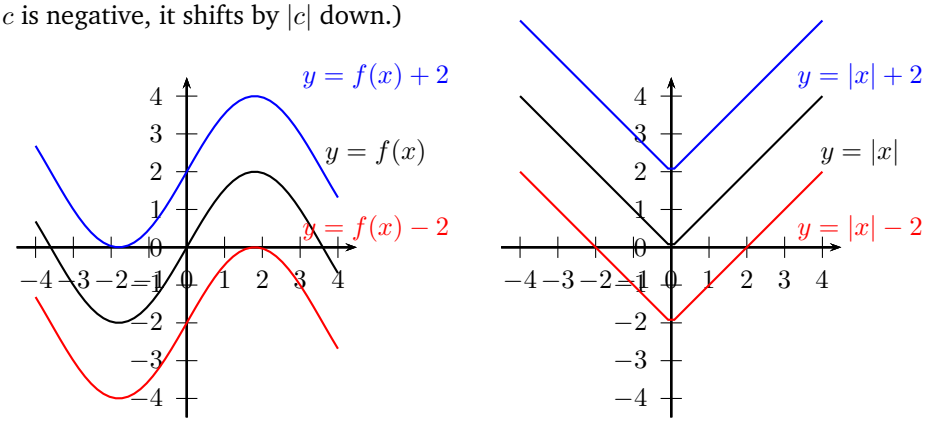
The figure below shows a graph of a function $y = \sqrt{x}$.



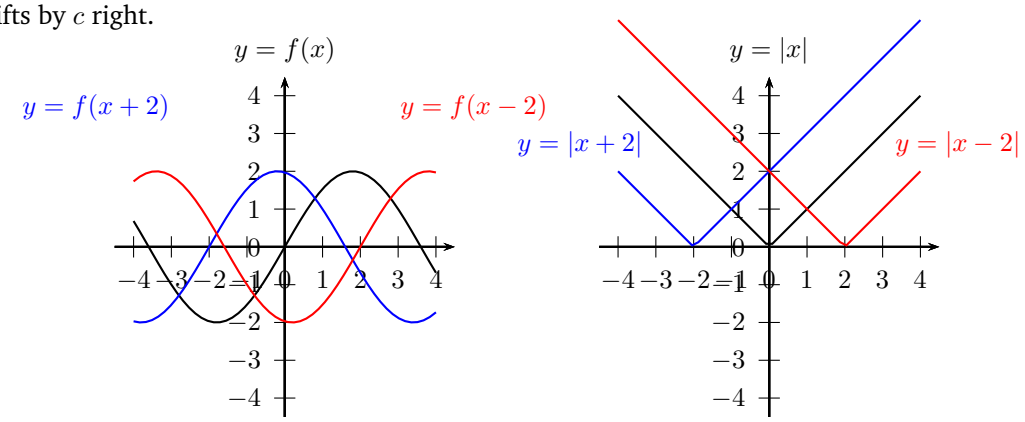
3. TRANSFORMATIONS

Having learned a number of basic graphs, we can produce new graphs, by doing certain transformations of the equations. Here are two of them.

Vertical translations: Adding constant c to the right-hand side of equation shifts the graph by c units up (if c is positive; if c is negative, it shifts by $|c|$ down.)



Horizontal translations: Adding constant c to x shifts the graph by c units left if c is positive; if c is negative, it shifts by c right.



HOMEWORK

1. (a) Sketch the graphs of functions $y = |x + 1|$ and $y = -x + 0.25$.
(b) How many solutions do you think this equation has?

$$|x + 1| = -x + 0.25$$

Note: you are not asked to find the solutions — just answer how many are there.

2. (a) Draw the graph of the equation $x^2 + y^2 - 1 = 0$.
(b) Draw the graph of the equation $x^2 + (y - 1)^2 - 1 = 0$.
(c) Draw the graph of the equation $xy = 0$.
(d) Draw the graph of the equation $x^2 + y^2 = 0$.

3. Sketch graphs of the following functions:

(a) $y = |x| + 1$ (b) $y = |x + 1|$ (c) $y = |x - 5| + 1$

4. Sketch graphs of the following functions:

(a) $y = x^2 + 3$ (b) $y = (x - 4)^2 - 1$ (c) $y = \sqrt{x + 3} + 1$

***5.** Sketch the following functions:

(a) $y = |x| + |x + 1|$ (b) $y = |x - 1| + |x + 1|$ (c) $|y| = x$

[Hint: Do draw graphs for (a) and (b), draw the graph of each of the summands, and then try to add the graphs