

MATH 7: HANDOUT 19
COORDINATE GEOMETRY 1: REVIEW. LINES AND CIRCLES. BASIC TRANSFORMATIONS

1. COORDINATE GEOMETRY: INTRODUCTION

In this section of the course we are going to study coordinate geometry. The basic notion is the **coordinate plane** – a plane with a given fixed point, called the **origin**, as well as two perpendicular lines – **axes**, called the ***x*-axis** and the ***y*-axis**. *x*-axis is usually drawn horizontally, and *y*-axis — vertically. These two axes have a **scale** – “distance” from the origin.

The scales on the axes allow us to describe any point on the plane by its **coordinates**. To find coordinates of a point *P*, draw lines through *P* perpendicular to the *x*- and *y*-axes. These lines intersect the axes in points with coordinates x_0 and y_0 . Then the point *P* has *x*-coordinate x_0 , and *y*-coordinate y_0 , and the notation for that is: $P(x_0, y_0)$.

The **midpoint** *M* of a segment *AB* with endpoints $A(x_1, y_1)$ and $B(x_2, y_2)$ has coordinates:

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

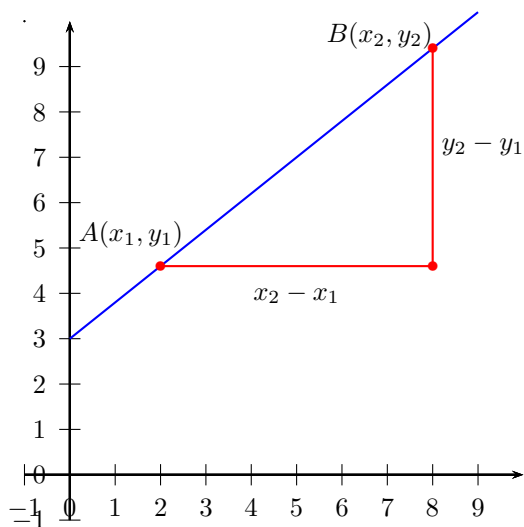
2. LINES

Given some relation which involves variables *x*, *y* (such as $x + 2y = 0$ or $y = x^2 + 1$), we can plot on the coordinate plane all points $M(x, y)$ whose coordinates satisfy this equation. Of course, there will be infinitely many such points; however, they usually fill some smooth line or curve. This curve is called the **graph** of the given relation.

Every relation (**equation**) of the form:

$$y = mx + b$$

where *m*, *b* are some numbers, defines a straight line. The slope of this line is determined by *m*: as you move along the line, *y* changes *m* times as fast as *x*, so if you increase *x* by 1, then *y* will increase by *m*:



In other words, given two points $A(x_1, y_1)$ and $B(x_2, y_2)$ **slope** can be computed by dividing change of *y*: $y_2 - y_1$ by the change of *x*: $x_2 - x_1$:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Two non vertical lines are **parallel** if and only if they have the **same slope**.

In the equation $y = mx + b$, *b* is a ***y*-intercept**, and determines where the line intersects the vertical axis (*y*-axis).

The equation of the **vertical** line is $x = k$, and the equation of the **horizontal** line is $y = k$. Notice that in case of the vertical line, the slope is undefined.

3. DISTANCE BETWEEN POINTS. CIRCLES

The distance between two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is given by the following formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

This formula is a straightforward consequence of the Pythagoras' Theorem.

HOMEWORK

1. A point B is 5 units above and 2 units to the left of point $A(7, 5)$. What are the coordinates of point B ?
2. Find the coordinates of the midpoint of the segment AB , where $A(3, 11)$, $B(7, 5)$.
3. Draw points $A(4, 1)$, $B(3, 5)$, $C(-1, 4)$. If you did everything correctly, you will get 3 vertices of a square. What are coordinates of the fourth vertex? What is the area of this square?
4. 3 points $(0, 0)$, $(1, 3)$, $(5, -2)$ are the three vertices of a parallelogram. What are the coordinates of the remaining vertex?
5. Consider the triangle $\triangle ABC$ with the vertices $A(-2, -1)$, $B(2, 0)$, $C(2, 1)$. Find the coordinates of the midpoint of B and C . Find the length of the median (i.e. a median connects a vertex with the midpoint of the opposite side) from A in the triangle $\triangle ABC$.
6. In this problem you will find equations that describe some lines.
 - (a) What is the equation whose graph is the y -axis?
 - (b) What is the equation of a line whose points all lie 5 units above the x -axis?
 - (c) Is the graph of $y = x$ a line? Draw it.
 - (d) Find the equation of a line that contains the points $(1, -1)$, $(2, -2)$, and $(3, -3)$.
7. For each of the equations below, draw the graph, then draw the perpendicular line (going through the point $(0, 0)$) and then write the equation of the perpendicular line

$$(a) y = 2x \quad (b) y = 3x$$

$$(c) y = -x \quad (d) y = -\frac{1}{2}x$$

Can you determine the general rule: if the slope of a line is k , what is the slope of the perpendicular line?

8. Find the equation of the line through $(1, 1)$ with slope 2.
9. Find the equation of the line through points $(1, 1)$ and $(3, 7)$. [Hint: what is the slope?]
10.
 - (a) Find k if $(1, 9)$ is on the graph of $y - 2x = k$. Sketch the graph.
 - (b) Find k if $(1, k)$ is on the graph of $5x + 4y - 1 = 0$. Sketch the graph.
11. Let l_1 be the graph of $y = x + 1$, l_2 be the graph of $y = x - 1$, m_1 be the graph of $y = -x + 1$, and m_2 be the graph of $y = -x - 1$.
 - (a) Find the intersection point of l_1 and m_1 ; Label this point P and write down its coordinates.
 - (b) Find the intersection point of l_2 and m_2 ; Label this point P and write down its coordinates.
 - (c) Find the midpoint of AB and write down its coordinates.
 - (d) Let C be the intersection point of l_1 with m_2 , and D be the intersection point of l_2 with m_1 . What kind of quadrilateral is $ABCD$?
 - (e) Explain why l_1 and l_2 are parallel. What is the distance between them?
12. Find the intersection point of a line $y = x - 3$ and a line $y = -2x + 6$. Sketch the graphs of these lines.