MATH 7: HANDOUT 13
QUADRATIC EQUATIONS 1

QUADRATIC EQUATIONS.

Today we discussed how one can approach solving quadratic equation:

$$ax^2 + bx + c = 0$$

The method used is called "completing the square". Here is an example how it works:

$$x^2 + 6x + 2 = x^2 + 2 \cdot 3x + 9 - 7 = (x+3)^2 - 7 = (x + 3 + \sqrt{7})(x + 3 - \sqrt{7})$$

thus, $x^2 + 6x + 2 = 0$ if and only if $x + 3 + \sqrt{7} = 0$, which gives $x = -3 - \sqrt{7}$, or $x + 3 - \sqrt{7} = 0$, which gives $x = -3 + \sqrt{7}$.

Note that in solving such equations we utilize the following important formulas we talked about earlier:

$$(x+a)^2 = x^2 + 2ax + a^2$$
$$(x-a)^2 = x^2 - 2ax + a^2$$
$$a^2 - b^2 = (a-b)(a+b)$$

HOMEWORK

1. Solve the equation $(x-1)^2 = 6$

2. Solve the following equations. Carefully write all the steps in your argument.

(a) $x^2 - 6x + 5 = 0$
(b) $x^2 - 5x + 5 = 0$
(c) $x^2 = 1 + x$
(d) $x^2 + 10x + 16 = 0$
(e) $x^2 + 8x + 16 = 0$
(f) $x^2 - 7x + 11 = 0$
(g) $x^2 + 3x + 1 = 0$
(h) $x^3 + 4x^2 - 45x = 0$

3. Indian mathematicians were aware of the quadratic formula for solving quadratic equations. Can you solve the following problem by the 9th century mathematician Mahāvīra?

One-third of a herd of elephants and three times the square root of the remaining part of the herd were seen on a mountain slope; and in a lake was seen a male elephant along with three female elephants constituting the ultimate remainder. How many were the elephants here?