MATH 7: HANDOUT 6
ARITHMETIC SEQUENCES. GEOMETRIC SEQUENCES.

Sum of Arithmetic Sequences

Sometimes we might need to calculate the sum of the first n elements in the arithmetic sequence. The following formula is used for this purpose:

$$S_n = a_1 + a_2 + a_3 + \dots + a_n = n \times \frac{a_1 + a_n}{2}$$

Proof: To prove this, we write the sum in 2 ways, in increasing and decreasing order:

$$S_n = a_1 + a_2 + \dots + a_n$$
$$S_n = a_n + a_{n-1} + \dots + a_1$$

Adding these two expressions up and noticing that $a_1 + a_n = a_2 + a_{n-1} = a_3 + a_{n-2} = \dots$ we get:

$$2S_n = (a_1 + a_n) \times n$$
$$S_n = n \times \frac{a_1 + a_n}{2}$$

Geometric Sequences

A sequence of numbers is a **geometric sequence** or **geometric progression** if if the next number in the sequence is the current number times a fixed constant called the **common ratio** or q.

Example: The sequence $6, 12, 24, 48, \dots$ is a geometric sequence because the next number is obtained from the previous by multiplication by $q = 2$.

We can also find the n-th term if we know the 1st term and q.

Example: What is a_{10} in the example above?

$$a_1 = 6$$
$$a_2 = a_1 q = 6 \cdot 2 = 12$$
$$a_3 = a_2 q = (a_1 q) q = a_1 q^2 = 6 \cdot 2^2 = 24$$

The pattern is:

$$a_n = a_1 q^{n-1}$$
$$a_{10} = a_1 q^9 = 6 \cdot 2^9 = 6 \cdot 512 = 3072$$

Properties of a Geometric Sequence. Any term is the **geometric mean** of its neighbors:

$$a_n = \sqrt{a_{n-1} \cdot a_{n+1}}$$

Proof:

$$a_n = a_{n-1} q$$
$$a_n = a_{n+1}/q$$

Multipluying these two equalities gives us:

$$a_n^2 = a_{n-1} \cdot a_{n+1}$$

from where we can get what we need. $\square$

HOMEWORK

1. Find the sum of the first 10 terms for the series: $4, 7, 10, 13, \ldots$
2. Find the sum of the first 1000 odd numbers.
3. Find the sum $2 + 4 + \cdots + 2018$.
4. There are 25 trees at equal distances of 5 meters in a line with a well, the distance of the well from the nearest tree being 10 meters. A gardener waters all trees separately starting from the well and he returns to the well after watering each tree to get water for the next. Find the total distance the gardener will cover in order to water all the trees.
5. An arithmetic progression has first term $a_1 = a$ and common difference $d = -1$. The sum of the first n terms is equal to the sum of the first $3n$ terms. Express a in terms of n.
6. * The sum of the first 20 terms of an arithmetic progression is 200, and the sum of the next 20 terms is -200. Find the sum of the first hundred terms of the progression.
7. Write the first 5 terms of a geometric progression if $a_1 = -20$ and $q = 1/2$.
8. What are the first two terms of the geometric progressions $a_1, a_2, 24, 36, 54, \ldots$?
9. Find the common ratio of the geometric progressions $1/2, -1/2, 1/2, \ldots$. What is a_{10}?
10. A geometric progression has 99 terms, the first term is 12 and the last term is 48. What is the 50th term?