Class Test
Nov. 10, 2024

1. Arithmetic sequences -summary

$$a_n = a_1 + (n-1)d$$
$$a_n = \frac{a_{n-1}+a_{n+1}}{2}$$
$$d = \frac{a_s - a_t}{s-t}$$
$$S = \frac{(a_1 + a_n) \times n}{2}$$

2. Geometric sequences -summary

$$a_n = a_1 \times q^{n-1}$$
$$a_n = \sqrt{a_{n-1}.a_{n+1}}$$
$$S_n = a_1 \times \frac{(1-q^n)}{1-q}$$
$$S = \frac{a_1}{1-q}$$

Problems

1. Factorize: (2.5 points each)
   a. $(x-3)^2 - (y+3)^2$
   b. $m^2 - m - 12$
   c. $a^4 - 144b^{10}$
   d. $x^4 + 4$

2. Factorize: (5 points each)
   a. $22x^2 + 47x + 21$
   b. $p^4 + 4z^{4n}$

3. Solve for x: (5 points each)
   a. $x^2 + 5x = 11/4$
   b. $4m^2 - 49p^2q^2 = 0$

10 points for the rest of the problems.

4. In the arithmetic progression 5, 17, 29, 41, . . . what term has a value of 497?

5. The 3-rd term of the arithmetic progression is equal to 1. The 10-th term of it is three times as much as the 6-th term. Find the first term and the common difference.
6. Write the first 5 terms of a geometric progression if $a_1 = -25$ and $q = \frac{1}{3}$
7. What are the first 2 terms of the geometric progression: a_1, a_2, 24, 36, 54, ...?
8. What is the common ratio of the geometric progression: $\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \ldots$? What is a_{10}? What is a_{100}?
9. Calculate the sum of infinite series: $\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \cdots$
10. What is the sum of : $1 - 2 + 2^2 - 2^3 + 2^4 - 2^5 + \cdots - 2^{15}$?
11. What is the sum of: $1 + x + x^2 + x^3 + x^4 + x^5 + \cdots + x^{100}$?
12. A geometric progression has 99 terms, the first term is 12 and the last term is 48. What is the 50-th term?