Homework 3: Algebraic identities, simplification and Pythagoras theorem.

HW is Due Oct 20th

Basic algebraic identities for refreshing your memory:

Exponents Laws

If a *and* b are real numbers and n is a positive integer

$(ab)^n = a^n b^n$ (eq. 1)

$\sqrt{ab} = \sqrt{a}\sqrt{b}$ (eq. 2)

$(a+b)^2 = a^2 + 2ab + b^2$ (eq.3)

$(a-b)^2 = a^2 - 2ab + b^2$ (eq.4)

And also: $a^2 - b^2 = (a-b)(a+b)$ (eq. 5)

Replacing in the last equality ***a*** by $\sqrt{\boldsymbol{a}}$, **b** by $\sqrt{\boldsymbol{b}}$, we get : $a - b = (\sqrt{a} - \sqrt{b})(\sqrt{a} + \sqrt{b})$ (eq. 6)

Also:

$$a^3 + b^3 = (a+b)\,(a^2 - ab + b^2)$$
$$a^3 - b^3 = (a-b)\,(a^2 + ab + b^2)$$

1. **Simplifying expressions with roots** (rational expressions)

The above identity (eq. 6) can be used to simplify expressions with roots by expanding the fractions with a term which "removes" the roots from the denominator:

$$\frac{1}{\sqrt{2}+1} = \frac{1}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1} = \frac{\sqrt{2}-1}{\left(\sqrt{2}\right)^2 - 1^2} = \frac{\sqrt{2}-1}{2-1} = \sqrt{2}-1$$

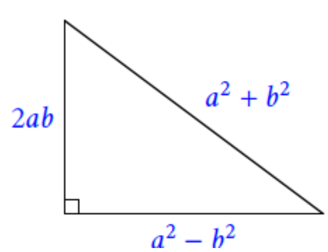
2. **Quadratic equations of a specific form**

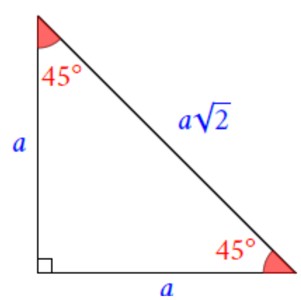
   We also discussed solving simple equations:
   - linear equation (i.e., equation of the form $ax + b = 0$, with a, b some numbers, and x the unknown and equation)
   - two types of quadratic equations (i.e, equations where the unknown is squared, x^2) when the left-hand side could be factored as product of linear factors, i.e, $(x-2)(x+3) = 0$.

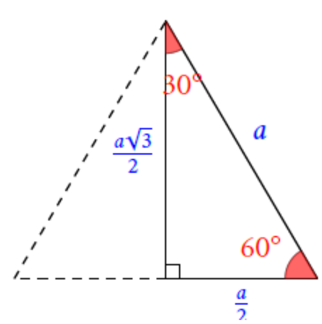
3. **Pythagoras' theorem:** We did not get time to discuss Pythagoras' theorem in the class today. The idea is not difficult and if you read the following paragraph, you should be able to all the homework problems related to this.

In a right triangle with legs ***a*** and ***b***, and hypotenuse ***c***, the square of the hypotenuse is the sum of squares of each leg: $c^2 = a^2 + b^2$. The converse is also true, if the three sides of a triangle satisfy $c^2 = a^2 + b^2$, then the triangle is a right triangle. Some Pythagorean triples are: (3,4,5), (5,12,13), (7,24,25), (8.15,17), (9,40,41), (11,60,61), (20,21,29).

To generate such Pythagorean triples, choose two positive integers a and b. Then plug the values into the sides as shown on the first picture:







Try to figure out again why the sides of this triangle satisfy the Pythagoras' Theorem!

45-45-90 Triangle: If one of the angles in a right triangle is 45^0, the other angle is also 45^0, and two of its legs are equal. If the length of a leg is *a*, the hypothenuse is $a\sqrt{2}$.

30-60-90 Triangle: If one of the angles in a right triangle is 30^0, the other angle is 60^0. Such triangle is a half of the equilateral triangle. That means that if the hypothenuse is equal to *a*, its smaller leg is equal to the half of the hypothenuse, i.e. $\frac{a}{2}$. Then we can find the other leg from the Pythagoras' Theorem, and it will be equal to $\frac{a\sqrt{3}}{2}$.

Homework problems

Instructions: Please always write solutions on a ***separate sheet of paper***. Solutions should include explanations. I want to see more than just an answer: I also want to see how you arrived at this answer, and some justification why this is indeed the answer. So **please include sufficient explanations**, which should be clearly written so that I can read them and follow your arguments.

1. Solve for x (i.e. find values of x that satisfies the following equations):
   a. $3x^2 - 20x = -32$
   b. $x^2 - x = 3/4$
   c. $4m^2 - 49p^2q^2 = 0$
   d. $x^2 + 5x = 11/4$
   e. d. $x^2 + 3x = -5/4$

2. Write each of the following expressions in the form $a + b\sqrt{3}$ with rational a, b. (No root in the denominator)
   a. $\frac{1}{1-2\sqrt{3}}$
   b. $\frac{1+\sqrt{3}}{1-\sqrt{3}}$
   c. $\frac{1+2\sqrt{3}}{\sqrt{3}}$
   d. $\frac{3-\sqrt{5}+\sqrt{7}}{3-\sqrt{5}-\sqrt{7}}$
   e. $\frac{2+4\sqrt{7}}{2\sqrt{7}-1}$
   f. $\frac{a-\sqrt{a-1}}{a+\sqrt{a-1}}$
   g. $\frac{\sqrt{p+q}-\sqrt{p-q}}{\sqrt{p+q}+\sqrt{p-q}}$

3. In a trapezoid ABCD with bases AD and BC, $\angle A = 90^0$, and $\angle D = 45^0$. It is also known that *AB* = 10 cm, and *AD* = 3*BC*. Find the area of the trapezoid.

4. In a right triangle ABC, BC is the hypotenuse. Draw AD perpendicular to BC, where D is on BC. The length of BC=13, and AB=5.What is the length of AD?