Homework 2: Algebraic identities, simplification and factorization.

HW is Due Oct 6th

Basic algebraic identities for refreshing your memory:

Exponents Laws

If a *and* b are real numbers and n is a positive integer

$(ab)^n = a^n b^n$ (eq. 1)

$\sqrt{ab} = \sqrt{a}\sqrt{b}$ (eq. 2)

$(a+b)^2 = a^2 + 2ab + b^2$ (eq.3)

$(a-b)^2 = a^2 - 2ab + b^2$ (eq.4)

And also: $a^2 - b^2 = (a-b)(a+b)$ (eq. 5)

Replacing in the last equality $\boldsymbol{a}$ by $\sqrt{\boldsymbol{a}}$, **b** by $\sqrt{\boldsymbol{b}}$, we get : $a - b = (\sqrt{a} - \sqrt{b})(\sqrt{a} + \sqrt{b})$ (eq. 6)

Also:

$$a^3 + b^3 = (a+b)\,(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a-b)\,(a^2 + ab + b^2)$$

Homework problems

Instructions: Please always write solutions on a ***separate sheet of paper***. Solutions should include explanations. I want to see more than just an answer: I also want to see how you arrived at this answer, and some justification why this is indeed the answer. So **please include sufficient explanations**, which should be clearly written so that I can read them and follow your arguments.

1. Simplify
   a) $\frac{1}{a}+\frac{1}{b}$
   b) $\frac{3}{x}+\frac{5}{xy}+\frac{5}{3a}$
   c) $\frac{x}{(x^2-y^2)}-\frac{y}{(x+y)^2}$
   d) $\frac{a}{b}+\frac{b}{c}+\frac{c}{d}$
   e) $\frac{a+b}{(b-c)(c-a)}+\frac{b+c}{(c-a)(a-b)}+\frac{c+a}{(a-b)(b-c)}$

2. Using algebraic identities calculate
   a. $299^2+598+1$
   b. 199^2
   c. $51^2-102+1$
   d. 16^3 - 15^3
   e. 57^3 - 56^3

3. Expand
   a. $(4a-b+c)^2=$
   b. $(a+9)(a-9)=$
   c. $(3a-2b)^4=$

4. Factorize (i.e., write as a product) the following expressions:
   a. $3p^2-20p+32$
   b. $21b^2-37b-28$
   c. $22x^2+47x+21$
   d. $27x^3-y^3$
   e. $8m^3-27p^3q^3$
   f. $24z^2-103z+55$
   g. a^4-169b^{10}
   h. $9z^4-148z^2+64$