Homework 1: Algebraic operations and factorization

HW1 is Due Sept 29

This week we reviewed various types of problems encountered in factorization and simplification of algebraic identities.
Some basic algebraic rules for you to remember (similar to last homework set):

1. **Exponents Laws**

If a is a real number, n is a positive integer $a^n = \underbrace{a \times a \times \cdots \times a}_{\text{n-times}}$

$$a^0 = 1$$

$$a^m \times a^n = a^{m+n}$$

$$a^m \div a^n = a^{m-n}$$

$$(ab)^n = a^n b^n$$

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

$$a^{-n} = \frac{1}{a^n}$$

$$(a^m)^n = a^{mn}$$

2. **Radicals**

$$a^{\frac{m}{n}} = \sqrt[n]{a^m}$$
$$\sqrt{ab} = \sqrt{a}\sqrt{b}$$

3. **Main Algebraic Identities**

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$a^2 - b^2 = (a-b)(a+b)$$

Homework problems on next page

Instructions: Please always write solutions on a ***separate sheet of paper***. Solutions should include explanations. I want to see more than just an answer: I also want to see how you arrived at this answer, and

some justification why this is indeed the answer. So **please include sufficient explanations**, which should be clearly written so that I can read them and follow your arguments.

1. Simplify
   a. $\sqrt{\frac{56}{13}} \cdot \sqrt{\frac{26}{7}} =$
   b. $\sqrt{48} =$
   c. $\frac{\sqrt{48}}{\sqrt{15}} =$

2. Express the following expressions in the form $2^r 3^s a^m b^n$:
   a. $8a^3 b^2 (27a^3)(2^5 ab) =$
   b. $3^2(2ab)^3(16a^2b^5)(24b^2a) =$
   c. $16a^2b^3(6ab^4)(ab^2)^3 =$

3. Expand as sums of powers of x:
   a. $(2x+5)^2 =$
   b. $(2-4x)^2 =$
   c. $(1-2x)^2 =$

4. Factor (i.e., write as a product) the following expressions:
   a. $4x^2 + 8xy + 4y^2$
   b. $9x^2 - 25$
   c. $(x-2)^2 - (y+3)^2$
   d. $256 - a^8 b^8$
   e. $(x-2)^2 - 10(x-2) + 25$
   f. $3x^3 - x^2y + 6x^2y - 2xy^2 + 3xy^2 - y^3$
   g. $a^2 - b^2 - 10b - 25$
   h. $x^4 + 4$
   i. $p^4 + 4z^{4n}$
   j. $a^2 + 3a + 2$
   k. $m^2 - m - 12$

5. Solve the following equations :
   a. $5(x+1) = 3x + 2$
   b. $(x^2-1)(x+2) = 0$
   c. $\frac{x+2}{x+3} = 2$
   d. $(x-3)(x+4) = 0$
   e. $x^2 + 4x = 0$
   f. $x^3 + 4x = 0$

6. Prove:
   a. $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
   b. $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$
   c. $Find (a+b)^4, (a-b)^4 \ using \ the \ previous \ results$