Math 6 revision: Algebraic operations expansion, factorization, formulas for a fast multiplication

1. **Exponents Laws**

If a is a real number, n is a positive integer

$$a^n = a \times a \times \cdots \times a$$

n-times

$$a^0 = 1$$

$$a^m \times a^n = a^{m+n}$$

$$a^m \div a^n = a^{m-n}$$

$$(ab)^n = a^n b^n$$

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

$$a^{-n} = \frac{1}{a^n}$$

$$(a^m)^n = a^{mn}$$

2. **Radicals**

$$a^{\frac{m}{n}} = \sqrt[n]{a^m}$$
$$\sqrt{ab} = \sqrt{a}\sqrt{b}$$

3. **Main Algebraic Identities**

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$a^2 - b^2 = (a-b)(a+b)$$

1. The difference between two numbers is $\frac{5}{12}$. If $\frac{3}{4}$ of the larger number is $\frac{3}{8}$ more than $\frac{1}{2}$ of the smaller, find the larger number.

2. Without a calculator, compute

$$19999 \cdot 20001$$

Is there a shorter way of doing it than the straightforward multiplication?

3. Expand the expressions:
   a. $(a^2 + 2a + 1)(a + 1)$
   b. $(2 - 4x)^2$
   c. $(4x - 7y)(4x + 7y)$
   d. $(6x^2 - y)(7x^2 - 2x - 5)$

4. Divide:
   a. $(b^2 - 2b + 1)/(b - 1)$
   b. $x^2 - 2x - yx + 2y\,/\,(\,x\text{-}2)$
   c. $(7x^2 - 2xy - 5y^2)/(x - y)$

5. Factor (i.e., write as a product) the following expressions:
   a. $x^2 + 3x^3$
   b. $x^2 - 2x - yx + 2y$
   c. $4x^2 - 4x + 1$
   d. $4x^2 + 16x + 2xy + 8y$
   e. $x^2(x + 4) + 5(x + 4)$
   f. $100x^8y^2 - 16x^4y^6$
   g. $a^2 + 4ab + 4b^2$
   h. $a^2 - 2a + 1$
   i. $x^2 - 7 \quad Hint\!: 7 = \left(\sqrt{7}\right)^2$
   j. $a^4 - b^4 \, Hint\!: a^4 = (a^2)^2$

6. John takes 15 min to walk from school to the bus station. Jim takes 20 min to walk from the school to the bus station. If the difference in their speeds is 2 km/h, how far is the station from the school?

7. Simplify:

$$a) \quad \frac{1}{(x + 1)} - \frac{1}{(x - 1)}$$

$$b) \quad \left(1 + \frac{1}{x}\right) \div (x + 1)$$

$$c) \quad \left(1 + \frac{1}{x}\right) \div \left(1 - \frac{1}{x}\right)$$