MATH 7 HOMEWORK 4: Algebraic identities. Pythagorean theorem
October 6, 2024

1. Exponents Laws

If a *and* b are real numbers and n is a positive integer

- **a.** $(ab)^n = a^n b^n$
- **b.** $\sqrt{ab} = \sqrt{a}\sqrt{b}$
- **c.** $(a+b)^2 = a^2 + 2ab + b^2$
- **d.** $(a-b)^2 = a^2 - 2ab + b^2$
- **e.** $a^2 - b^2 = (a-b)(a+b)$

Replacing in the last equality ***a*** by $\sqrt{a}$, **b** by $\sqrt{b}$, we get:

- **f.** $a - b = (\sqrt{a} - \sqrt{b})(\sqrt{a} + \sqrt{b})$

2. Simplifying expressions with roots (rational expressions)

The last identity above can be used to simplify expressions with roots by expanding the fractions with a term which "*removes*" the roots from the denominator:

$$\frac{1}{\sqrt{2}+1} = \frac{1}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1} = \frac{\sqrt{2}-1}{(\sqrt{2})^2 - 1^2} = \frac{\sqrt{2}-1}{2-1} = \sqrt{2}-1$$

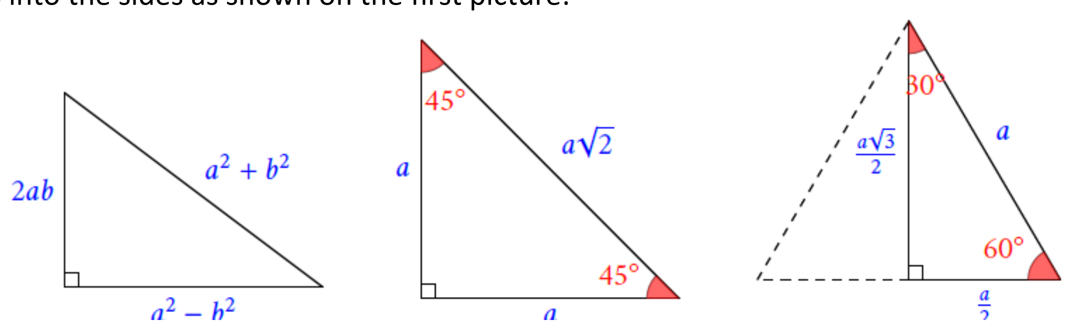
3. Quadratic equations of a specific form

- linear equation (i.e., equation of the form $ax + b = 0$, with a, b some numbers, and x the unknown and equation)
- two types of quadratic equations (i.e, equations where the unknown is squared, x^2) when the left-hand side could be factored as product of linear factors, i.e, $(x - 2)(x + 3) = 0$.

4. Pythagoras' theorem

In a right triangle with legs ***a*** and ***b***, and hypotenuse ***c***: $c^2 = a^2 + b^2$. The converse is also true, if the three sides of a triangle satisfy $c^2 = a^2 + b^2$, then the triangle is a right triangle. Some Pythagorean triples are: (3,4,5), (5,12,13), (7,24,25), (8.15,17), (9,40,41), (11,60,61), (20,21,29).

To generate such Pythagorean triples, choose two positive integers a and b. Then plug the values into the sides as shown on the first picture:



Try to figure out again why the sides of this triangle satisfy the Pythagoras' Theorem!

45-45-90 Triangle: If one of the angles in a right triangle is 45^0, the other angle is also 45^0, and two of its legs are equal.

If the length of a leg is *a*, the hypothenuse is $a\sqrt{2}$.

30-60-90 Triangle: If one of the angles in a right triangle is 30^0, the other angle is 60^0. Such triangle is a half of the

equilateral triangle. That means that if the hypothenuse is equal to *a*, its smaller leg is equal to the half of the hypothenuse, i.e. $\frac{a}{2}$. Then we can find the other leg from the Pythagoras' Theorem, and it will be equal to $\frac{a\sqrt{3}}{2}$.

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Instructions: Please always write solutions on a ***separate sheet of paper***. Solutions should include explanations **how you arrived at this answer.**

1. Simplify
   a. $\frac{6^3 \cdot 6^4}{2^3 \cdot 3^4} =$
   b. $(2^{-3} \cdot 2^7)^2 =$
   c. $\frac{3^2 \cdot 6^{-3}}{10^{-3} \cdot 5^2}$

2. Simplify
   a) $\frac{a}{2} + \frac{b}{4} =$
   b) $\frac{1}{a} + \frac{1}{b} =$
   c) $\frac{3}{x} + \frac{5}{xy} + \frac{5}{3a} =$

3. Solve system of equations:
   a. $\begin{cases} 6x - 5y = -3 \\ x + y = 5 \end{cases}$
   b. $\begin{cases} 5x + 2y = 16 \\ 2x + 3y = 13 \end{cases}$

4. Using algebraic identities calculate
   a. $299^2 + 598 + 1 =$
   b. $199^2 =$
   c. $51^2 - 102 + 1 =$

5. Expand
   a. $(4a - b)^2 =$
   b. $(a + 9)(a - 9) =$
   c. $(3a - 2b)^2 =$

6. Solve the following quadratic equations. *Hint: Factor first (i.e., write as a product)*:
   a. $x^2 - 18x + 81 = 0$
   b. $3x(x + 1) + 2(x + 1) = 0$
   c. $36a^2 - 49 = 0$

7. Write each of the following expressions in the form $a + b\sqrt{3}$ with rational a, b. (No root in the denominator):
   a. $\left(1 + \sqrt{3}\right)^2$
   b. $\left(1 + \sqrt{3}\right)^3$
   c. $\frac{1}{1-2\sqrt{3}}$
   d. $\frac{1+\sqrt{3}}{1-\sqrt{3}}$
   e. $\frac{1+2\sqrt{3}}{\sqrt{3}}$

8. In a trapezoid ABCD with bases AD and BC, $\angle A = 90^0$, and $\angle D = 45^0$. It is also known that AB = 10 cm, and AD = 3BC. Find the area of the trapezoid.

9. In a right triangle ABC, BC is the hypotenuse. Draw AD perpendicular to BC, where D is on BC. The length of BC=13, and AB=5.What is the length of AD?