

Math 6: Homework 2.2

Arithmetic + Geometric Sequences

Geometric Sequence

A sequence of numbers is a **geometric sequence** or **geometric progression** if the next number in the sequence is the current number times a fixed constant called the **common ratio** or q .

Example: The sequence 6,12,24,48,... is a geometric sequence because the next number is obtained from the previous by multiplication by $q = 2$.

We can also find the n -th term if we know the 1st term and q . **Example:** What is b_{10} in the sequence above?

$$b_1 = 6$$

$$b_2 = b_1q = 6 \cdot 2 = 12$$

$$b_3 = b_2q = (b_1q)q = b_1q^2 = 6 \cdot 2^2$$
 The

pattern is:

$$b_n = b_1 q^{n-1}$$

$$b_{10} = b_1q^9 = 6 \cdot 2^9 = 6 \cdot 512 = 3072$$

Arithmetic sequence reminder:

$$a_n = a_1 + (n - 1)d$$

$$\text{Sum: } S = a_1 + a_2 + a_3 + \dots + a_n = n \cdot \frac{a_1 + a_n}{2}$$

Problems:

1. What are the first two terms of the geometric progressions $b_1; b_2; 15; 18; 21,6; \dots$?
2. A geometric progression has 99 terms, the first term is 12 and the last term is 48. What is the 50th term?
3. You are taking part in a shuttle run competition: you need to run to the first marker and back, then to the second marker and back, etc. There are 8 markers at equal distances of 15 meters. The distance of the first marker from the start line being 20 meters. What is the total distance you will cover running in this competition?
4. An arithmetic progression begins with the terms 50, 47, 44, Will 0 be part of this arithmetic progression?
5. Find the sum $1 + 3 + 5 + \dots + 999 = ?$
6. John makes lemonade with a certain ratio of sugar to juice. If he doubles the sugar, the drink will have 10% sugar. What is the ratio of sugar to lemon juice in the original recipe?
7. *Let b_1, b_2, \dots, b_5 be a geometric sequence. Is it possible for these numbers to also form an arithmetic sequence? Are there any values of common ratio q that would be impossible (i.e., such numbers with common ratio q could never be an arithmetic sequence)?