MATH 6: HOMEWORK: COMBINATIONS

Permutations with Repetitions

If there are identical objects among the selections, then there will be some overcounting that we need to correct. We do that by dividing the number of arrangements of the repeated objects. Let's take a look at the number of ways to arrange the letters in the word WALL. If the letters would be distinct, then the number of arrangements (permutations) would be $4!$. But because the letter L is repeated, we need to divide by $2!$, the number of ways to arrange 2 objects (both L letters). The answer is $\frac{4!}{2!} = 12$. Another example how many different arrangements can be formed with the letters from the word ALLELE? The answer is: $\frac{6!}{3!2!} = 20$. We divide by 3! because the three letters L are indistinguishable, and 2! for the Es.

Combinations

Combinations are used when order does not matter. For example, when a sports team is selected, or a committee is formed, it does not matter which team member is selected first or second or third. What matters is only the fact that they are selected. We can think of this as arrangements that need to be corrected because of overcounting - essentially, everyone on the selected team is 'identical'. Combinations can be written as $\binom{n}{k}$ for k objects chosen out of n objects, we say "n choose k". Combinations can be also written as $_n C_k$. The two notations are equivalent.

$$_n\mathrm{C}_\mathrm{k} = \binom{\mathrm{n}}{\mathrm{k}} = \frac{\mathrm{n}!}{\mathrm{k}!(\mathrm{n}-\mathrm{k})!} = \frac{_\mathrm{n}\mathrm{P}_\mathrm{k}}{\mathrm{k}!}$$

1. (a) List all arrangements of letters in the word WALK.
   (b) Copy the same list again, except replace all the letters K with the letter L.
   (c) Cross out any repeated arrangements in your second list (that is, if you find two identical arrangements of letters, you may cross one out).
   (d) Explain why the number of ways to arrange the letters of WALK is double the number of ways to arrange the letters of WALL.

2. In how many ways can you select a team of 3 from 8 people?

3. A family has 4 sons and 3 daughters. In how many ways can they be seated on 7 chairs so that at least 2 boys are next to each other? (Hint: Use complement counting. In what case are there no boys sitting next to each other?)

4. Show that $\binom{n}{k} = \binom{n}{n-k}$

5. *You are given two indistinguishable envelopes, each of which contains some amount of money. It is known that one envelope contains 10 times as much as the other. You pick one envelope at random; it is opened, and you see that it contains $50. Now you are given a choice: do you want to keep this money — or do you want to take the other envelope instead?