

**MATH 6. BASIC PROBABILITY
HOMEWORK: LET'S PLAY CASINO!**

Basic probability

Basic probability rule:

$$P(\text{win}) = \frac{\text{number of winning outcomes}}{\text{total number of possible outcomes}}$$

(assuming all outcomes are equally likely!)

For example, probability of drawing a spade card out of the standard deck is

$$P = \frac{13}{52} = \frac{1}{4}$$

Complement rule

If probability of some event is P then the probability that this event will **not** happen is $1 - P$.

For example, if we draw a card from the deck then the probability that it is **not** a spade is

$$1 - \frac{1}{4} = \frac{3}{4}.$$

Product rule

If we do two trials (e.g., rolling a die twice), then the probability of getting result A in the first trial and result B in the second one is

$$P(A, \text{ then } B) = P(A)P(B)$$

if results of the second trial **do not depend** on the first one.

Example: tossing a coin

Question. If toss a coin 10 times, what is the probability that all will be heads?

Answer. $\left(\frac{1}{2}\right)^{10} = \frac{1}{2^{10}}$ (using calculator, one can compute that it is $1/1024 \approx 0.001$, or 1/10 of 1%).

1. a) What is the probability that if we roll 2 dice, the sum will be at most 7?
b) A and B are playing the following game. They roll 2 dice; if the sum is at most 7, A wins, and B pays him \$1. Otherwise A loses and he pays to B \$1.
Would you prefer to play for A or for B in this game?
2. a) What is the probability that if we roll 3 dice, all the numbers will be odd?
b) A and B are playing the following game. They roll 3 dice; if all numbers are odd, A wins, and B pays him \$5. Otherwise A loses and he pays \$1 to B.
Would you prefer to play for A or for B in this game?
3. Supposing that there are equal chances of a boy or a girl being born, what is the probability that the first five babies born next Saturday morning at the St. Charles Hospital will be girls? That at least one of them five will be a girl?
4. We take the standard card deck and draw one card. What is the probability that it will be:
 - a) '5' of spades
 - b) a queen
 - c) a jack of red color
 - d) a non-ace
 - e) neither a heart nor a jack
5. A fair 6-sided die is rolled twice. What is the probability that the first number is greater or equal than the second one?
6. a) In a class of 25 students, everyone chooses a date (e.g., March 13). How many combinations are possible? (Students only choose month and day, not year; February 29th is not allowed, so there are 365 different possibilities. Also, it matters who had chosen which day: combination where Bill has chosen March 12 and John, June 15 is considered different from the one where Bill has chosen June 15 and John March 12.)
 - *b) In the same situation, how many such combinations are possible if we additionally require that all dates must be different?
 - *c) Suppose now that each of these 25 students has chosen a date at random, not knowing the choices of others. What is the probability that all of these dates will be different? That at least 2 will coincide?