Logic and sets – Math Battle

1. You meet two inhabitants: Carl and Bill. Carl says, 'I and Bill are both knights or both knaves.' Bill claims, 'Only a knave would say that Carl is a knave.' Can you determine who is a knight and who is a knave?   (3 points)
2. If today is Thursday, then Jane's class has library day. If Jane's class has library day, then Jane will bring home new library books. Jane brought no new library books. Therefore,. . . (1 point)
3. Write truth tables for formulas:
    a) P ∪ notQ   (3 points)
    b) notP ∩ (q ∪ notR)   (3 points)
4. You meet two inhabitants: Homer and Bill. Homer tells you that it's not the case that Bill is a knave. Bill tells you, 'Homer and I are not the same.' Can you determine who is a knight and who is a knave?   (3 points)
5. You meet two inhabitants: Homer and Abe. Homer claims that it's false that Abe is a knave. Abe tells you, 'I could claim that Homer is a knave.' Can you determine who is a knight and who is a knave?   (3 points)
6. Find the greatest common divisor and least common multiple of 132 and 90.   (2 points)
7. (This question should be answered without using a calculator.) The maximal distance from Sun to Pluto is 7,375,927,931 km. Speed of light is about 300,000 km/sec. How long does it take for Sun's light to reach Pluto? (You do not need to give a precise answer — an approximate one like "about 2 minutes" would be fine.)   (2 points)
8. There are 30 students in a class. Among them, 8 students are learning both French and Spanish. A total of 18 students are learning French. If every student is learning at least one language, how many students are learning Spanish in total?   (3 points)
9. Using Venn diagrams, explain why
    a) $\overline{A \cap B} = \overline{A} \cup \overline{B}$.   (3 points)
    b) Do the same for formula A ∩ (B ∪ C) = (A ∩ B) ∪ (A ∩ C).   (3 points)

10. If A = {1, 3, 5}, B = {3, 5, 6} and C = {1, 3, 7}

    (a) Verify that A ∪ (B ∩ C) = (A ∪ B) ∩ (A ∪ C)   (3 points)

    (b) Verify A ∩ (B ∪ C) = (A ∩ B) ∪ (A ∩ C)   (3 points)

11. In a survey of university students, 64 had taken mathematics course, 94 had taken chemistry course, 58 had taken physics course, 28 had taken mathematics and physics, 26 had taken mathematics and chemistry, 22 had taken chemistry and physics course, and 14 had taken all the three courses. Find how many had taken one course only?   (5 points)

12. Let U be a universal set consisting of all the natural numbers until 20 and set A and B be a subset of U defined as A = {2, 5, 9, 15, 19} and B = {8, 9, 10, 13, 15, 17}. Find A ∪ B.   (2 points)

13. Find the union of sets, A = {x : x is a natural number and multiple of 3} and B = {x : x is a natural number less than 6}   (3 points)

14. A barber in a small town decides that he will shave all men who do not shave themselves (and only them). Should he shave himself?   (2 points)