MATH 6
HANDOUT 5: SETS

SETS

By word *set*, we mean any collection of objects: numbers, letters,… Objects of the set are usually referred to as *elements* of this set.

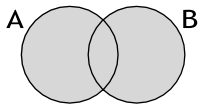
Members of sets. Sometimes we might have to say whether the element belongs to the set or not. In this case the following notation is used:

- $x \in A$ means "x is in A", or "x is an element of A"
- $x \notin A$ means "x is not in A"

Set Operations. There are several operations that can be used to get new sets out of the old ones:

- $A \cup B$: *union* of A and B. It consists of all elements which are in either A or B (or both):

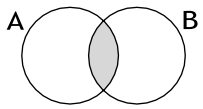
$$A \cup B = \{x \in A \text{ OR } x \in B\}.$$



Pictures with intersecting circles like this are called Venn diagrams.

- $A \cap B$: *intersection* of A and B. It consists of all elements which are in both A and B:

$$A \cap B = \{x \in A \text{ AND } x \in B\}.$$



- $\overline{A}$: *complement* of A, i.e. the set of all elements which are not in A: $\overline{A} = \{x \notin A\}$.

Intervals. The following notations are used when we talk about intervals on the number line. Intervals may have end points included or excluded: [and] represent that the end point is included, while (and) indicate that the end point is excluded.

$[a, b] = a \le x \le b$ is the interval from a to b (including endpoints),
$(a, b) = a < x < b$ is the interval from a to b (**not** including endpoints),
$[a, \infty) = a \le x$ is the half-line from a to infinity (including a),
$(a, \infty) = a < x$ is the half-line from a to infinity (**not** including a)

Homework

1. If Al comes to a party, Betsy will not come. Al never comes to a party where Charley comes. And either Betsy or Charley (or both) will certainly come to the party.
   Based on all of this, can you explain why it is impossible that Al comes to the party?
2. Let us take the usual deck of cards. As you know, there are 4 suits, hearts, diamonds, spades and clubs, 13 cards in each suit.
   Denote:
   - H=set of all hearts cards
   - Q=set of all queens
   - *R*=set of all red cards

   Describe by formulas (such as $H \cap Q$) the following sets:
   - all red queens
   - all black cards
   - all cards that are either hearts or a queen
   - all cards other than red queens

   How many cards are there in each set?

3. Let $A = [1, 3] = \{1 \le x \le 3\}$, $B = \{x \ge 2\}$, $C = \{x \le 1.5\}$.

   Describe $\overline{A}$, $A \cap B$, $A \cap C$, $A \cap (B \cup C)$, $A \cap (B \cap C)$.

4. For each of the sets below, draw it on the number line and then describe its complement:

   (a) $[0, 2]$ (b) $(-\infty, 1] \cup [3, \infty)$ (c) $(0, 5) \cup (2, \infty)$

5. In a group of people, 20 like milk, 30 like tea, 22 like coffee, 12 like coffee only, 6 like milk and coffee only, 2 like tea and coffee only and 8 like milk and tea only. Show this information in a Venn-diagram and find:

   (a) How many like at least one drink?

   (b) How many like exactly one drink?

***6.** In this problem, we denote by $|A|$ the number of elements in a finite set A.

   (a) Show that for two sets A, B, we have $|A \cup B| = |A| + |B| - |A \cap B|$.

   (b) Can you come up with a similar rule for three sets? That is, write a formula for $|A \cup B \cup C|$ which uses $|A|$, $|B|$, $|C|$, $|A \cap B|$, $|A \cap C|$, $|B \cap C|$, $|A \cap B \cap C|$.