MATH 6
TRUTH TABLES AND MORE LOGIC OPERATORS

Basic logic operations:

· NOT A: true if A is false, false if A is true
· A AND B: true if both A and B are true, false otherwise
· A OR B: true if at least one of A and B is true, false otherwise
· A XOR B: true if exactly one of A and B is true, false otherwise
· A NAND B = NOT (A AND B): false if both A and B are true, true otherwise

Operation ⇒ (reads "implies", or "if A **then** B**).** Here are some of the more important rules:
· A ⇒ B and B ⇒ A are not equivalent: it is possible that one statement is true and the other is false.

Logic gates and computer chips

Computer chips are using logical operations: each of the inputs and outputs can have voltage 0 or some positive voltage. The usual convention is

· Positive voltage (1) = true
· Zero voltage (0) = false

1. A certain convention numbered 100 politicians. Each politician was either crooked or honest. We are given the following two facts:
At least one of the politicians was honest. Given any two of the politicians, at least one of the two was crooked.
Can it be determined from these two facts how many of the politicians were honest and how many of them were crooked?

2. A particular musical elephant enjoys dancing, but only if it is wearing purple. Observing this elephant, I take the following notes:
D = the elephant is dancing
P = the elephant is wearing purple
The elephant dances only when wearing purple. It sometimes naps, no matter the color it is wearing.
From this, conclude whether the following statements are true or false:

(a) D ⟹ P
(b) P ⟹ D

3. You probably know Lewis Carroll as the author of *Alice in Wonderland* and other books. What you might not know is that he was also a mathematician very much interested in logic, and had invented a number of logic puzzles. Here is one of them. You are given 3 statements.
(a) *All babies are illogical.*
(b) Nobody is despised who can manage a crocodile.
(c) Illogical persons are despised.
Can you guess what would be the natural conclusion from these 3 statements?

4. Consider the circuit below: there are two NAND chips, and output of each one is connected to the input of the other. If inputs A, B are both true (i.e., carry positive voltage), what will be the values of X, Y?