MATH 6: HOMEWORK 13
COORDINATE GEOMETRY: EQUATION OF A CIRCLE

Equation of a line. Last week, we learned that we can express a line in the coordinate plane through the following linear equation

$$y = mx + b,$$

where m stands for the slope of the line and b stands for the y-intercept.

If we know two points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ through which the line passes, we can find the slope m by doing

$$m = \frac{y_1 - y_2}{x_1 - x_2}.$$

Midpoint of a segment: If we have two points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$, the midpoint of the segment connecting these two points is given by

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$

Parallel and perpendicular lines: Parallel lines are defined by having the same slope $m_1 = m_2$. In perpendicular lines, the slopes of the two lines are related by $m_1 = -1/m_2$.

Distance between two points: In order to find the distance between two points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ in the coordinate plane, we can make use of the Pythagorean theorem.

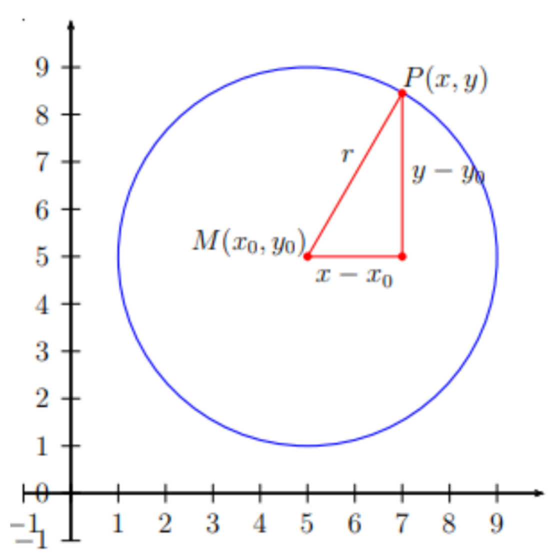
If we connect the two points by a straight line, then we can make a right triangle where this line is the hypotenuse. The legs will then be a horizontal and a vertical line. From this, we can use the Pythagorean theorem to find that the distance d between the two points is

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}.$$

Equation of a circle: A circle is defined as all the points in the coordinate plane that have equal distance to a fixed point $M(x_0, y_0)$. The distance to this point is what we call the RADIUS of the circle. The equation that describes a circle centered at $M(x_0, y_0)$ with a radius of r is given by

$$(x - x_0)^2 + (y - y_0)^2 = r^2.$$

Can you see any similarity with the equation of the distance between two points?



HOMEWORK

1. Find the equation of the line through $(1,1)$ with slope 2.

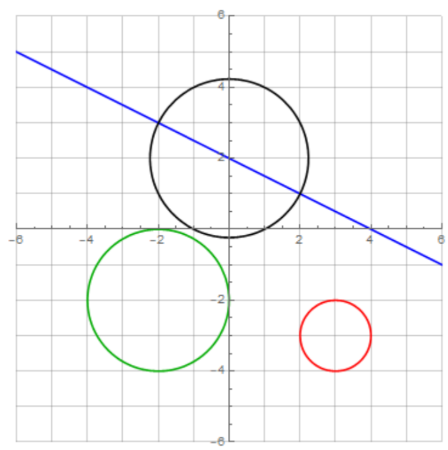
2. Find the equation of the line through points $(1,1)$ and $(3,7)$.

3. Consider the following system of linear equations:

$$\begin{cases} 6x - 5y = -3 \\ x + \ \ y = 5 \end{cases}$$

(a) Solve the system of linear equations to find a value for x and y.
(b) Rewrite each of the equations in the form of $y = mx + b$.
(c) Graph each of these lines in a coordinate plane and find their intersection point.
(d) What can you say about the intersection point and the system of linear equations?

4. Find the equation of the circles and the line in the figure below. The black circle has a radius of $\sqrt{5}$.



***5.** Using your answer to the previous question, solve the following system of **non-linear** equations:

$$\begin{cases} y \quad = -x/2 + 2 \\ 5 = x^2 + (y-2)^2 \end{cases}$$

What does the solution represent?

6. Let l_1 be the graph of $y = x + 1$, l_2 be the graph of $y = x - 1$, m_1 be the graph of $y = -x + 1$, and m_2 be the graph of $y = -x - 1$.
(a) Find the intersection point of l_1 and m_1; Label this point A and write down its coordinates.
(b) Find the intersection point of l_2 and m_2; Label this point B and write down its coordinates.
(c) Find the midpoint of AB and write down its coordinates.
(d) Let C be the intersection point of l_1 with m_2, and D be the intersection point of l_2 with m_1. What kind of quadrilateral is $ABCD$?
(e) Are l_1 and l_2 parallel? Explain why or why not?

7. (a) Draw the graph of the equation $x^2 + y^2 - 1 = 0$.
(b) Draw the graph of the equation $x^2 + (y-1)^2 - 1 = 0$.
(c) Draw the graph of the equation $(x+2)^2 + (y+3)^2 = 4$.