

Math 6: Homework 12

Coordinate Geometry

In this section of the course, we are going to study coordinate geometry. The basic notion is the **coordinate plane** – a plane with a given fixed point, called the **origin**, as well as two perpendicular lines – **axes**, called the **x-axis** and the **y-axis**.

The midpoint

The midpoint M of a segment AB with endpoints $A(x_1, y_1)$ and $B(x_2, y_2)$ has coordinates:

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Straight line

This curve is called the **graph** of the given relation.
Every relation (**equation**) of the form:

$$y = mx + b$$

where m , b are some numbers, defines a **straight line**. The **slope** of this line is determined by m : as you move along the line, y changes m times as fast as x , so if you increase x by 1, then y will increase by m . And b is a **y-intercept**, it determines where the line intersects the vertical axis (y -axis).

In other words, given two points $A(x_1, y_1)$ and $B(x_2, y_2)$ **slope** can be computed by dividing change of y : $y_2 - y_1$ by the change of x : $x_2 - x_1$:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Two non-vertical lines are **parallel** if and only if they have the **same slope**.

Perpendicular lines slope give -1 if multiplied. Given any two lines k and l in the coordinate plane, let their slopes be u and v . If $k \perp l$, then $uv = -1$. On the other hand, if $uv = -1$, then $k \perp l$.

The distance between two points

$P(x_1, y_1)$ and $Q(x_2, y_2)$ is given by the following formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

This formula is a straightforward consequence of the Pythagoras' Theorem.

Homework Problems

Solve it on a separate piece of paper. Use quadrille paper to plot graph.

- 3 points $A(0,0)$, $B(1,3)$, $D(5,-2)$ are vertices of a parallelogram $ABCD$. What are the coordinates of point C ?
- In this problem you will find equations that describe some lines.
 - What is the equation whose graph is the y -axis?
 - What is the equation of a line whose points all lie 5 units above the x -axis?
 - Is the graph of $y = x$ a line? Draw it.
 - Find the equation of a line that contains the points $(1,-1)$, $(2,-2)$, and $(3,-3)$.
- For each of the equations below, draw the graph, then draw the perpendicular line (going through the point $(0, 0)$) and then write the equation of the perpendicular line:

$$(a) y = 3x$$

$$(b) y = -1/2x$$

- Find the equation of the line through $(1,1)$ with slope 2.
- Find the equation of the line through points $(1,1)$ and $(3,7)$. [Hint: what is the slope?]
- Find the intersection point of a line $y = x - 3$ and a line $y = -2x + 6$. Sketch the graphs of these lines.
- Find the area of a triangle with vertices at $(5,4)$, $(0,3)$, $(-1,-2)$.
 - Show that the quadrilateral with the vertices at $(-1,-2)$, $(4,-1)$, $(5,4)$, $(0,3)$ is a rhombus. Then, find its area.
- Sketch graphs of the following functions:

$$(a) y = |x| + 1$$

$$(b) y = |x + 1|$$

$$(c) y = |x - 5| + 1$$