MATH 6
HW 11: SETS CONTINUED

Counting

We denote by $|A|$ the number of elements in a set A (if this set is finite). For example, if $A = \{a, b, c, \ldots, z\}$ is the set of all letters of English alphabet, then $|A| = 26$.

If we have two sets that do not intersect, then $|A \cup B| = |A| + |B|$: if there are 13 girls and 15 boys in the class, then the total is 28.

If the sets do intersect, the rule is more complicated:

$$|A \cup B| = |A| + |B| - |A \cap B|$$

$\overline{A}$: complement of A, i.e. the set of all elements which are not in A: $\overline{A} = \{x \mid x \notin A\}$.

$|A|$: number of elements in a set A (if this set is finite).

Subsets. We say that set A is a subset of B (notation: $A \subset B$) if every element of A is also an element of B: $x \in A \Rightarrow x \in B$. Note that A can be equal to B.

1. Let $A = [1, 3] = \{x \mid 1 \leq x \leq 3\}$, $B = \{x \mid x \geq 2\}$, $C = \{x \mid x \leq 1.5\}$. Draw on the number line the following sets: $\overline{A}$, $\overline{B}$, $\overline{C}$, $A \cap B$, $A \cap C$, $A \cap (B \cup C)$, $A \cap B \cap C$.
2. A **subset** of a set A is a set formed by taking some (possibly all) elements of A; for example, the set $\{2, 4, 6, 8\}$ is a subset of the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$.
   List all subsets of the set $S = \{1, 2, 3\}$ (do not forget the empty set which contains no elements at all and S itself).
   Can you guess the general rule: if set S has n elements, how many subsets does it have?
3. (a) Using Venn diagrams, explain why $\overline{A \cap B} = \overline{A} \cup \overline{B}$. Does it remind you of one of the logic laws we had discussed before?
   (b) Do the same for formula $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.
4. In this problem, we denote by $|A|$ the number of elements in a finite set A.
   (a) Show that for two sets A, B, we have $|A \cup B| = |A| + |B| - |A \cap B|$.
   *(b) Can you come up with a similar rule for three sets? That is, write a formula for $|A \cup B \cup C|$ which uses $|A|, |B|, |C|, |A \cap B|, |A \cap C|, |B \cap C|$.
5. Consider the following sets:
   $\mathbb{Z}$ — all whole numbers (positive and negative)
   $\mathbb{N}$ — all positive whole numbers
   $\mathbb{R}$ — all numbers
   $\mathbb{Q}$ — all rational numbers (i.e., those that can be written as a fraction)
   Order them from smallest to largest, so that each set is a subset of the next one.
6. Find A if you know that $A \cup \{5, 7\} = \{3, 5, 7, 8\}$, $A \cap \{1, 2, 5, 7\} = \{5, 7\}$
7. In a class of 33 students, 12 are girls, 10 play soccer, and 10 play chess. Moreover, it is known that 6 of the soccer players are girls, that 2 of the chess players also play soccer, and that there is exactly one girl who plays both chess and soccer. Finally, 4 girls play neither soccer nor chess. Can you figure out how many boys play soccer? chess? both? neither?
8. 150 people at a Van Halen concert were asked if they knew how to play piano, drums or guitar.

(a) 18 people could play none of these instruments.
(b) 10 people could play all three of these instruments.
(c) 77 people could play drums or guitar but could not play piano.
(d) 73 people could play guitar.
(e) 49 people could play at least two of these instruments.
(f) 13 people could play piano and guitar but could not play drums.
(g) 21 people could play piano and drums.

How many people can play piano? drums?