Homework 3: Arithmetic sequences

HW4 is Due Oct 20th.

1. Arithmetic sequence (progression)

A sequence of numbers is an arithmetic sequence if the difference between consecutive elements is the same number. This number is called a common difference, d.

For example: 1, 5, 9, 13, 17, ... The difference here is d = 4.

Sequence elements (terms) are labeled according to their position in the sequence using a counter **n** as a subscript. The value of the n-th element in a sequence is labeled as a_n. Then, the first term in the sequence has n = 1 and a value of $a_1 = 1$, the second element is $a_2 = 5$, and so on.

We could find any element of a sequence knowing the first element a_1 and the difference d.
For example, what is a_{100}?

$a_1 = 1$
$a_2 = a_1 + d = 1 + 4 = 5$
$a_3 = a_2 + d = a_1 + 2d = 1 + 2 \times 4 = 9$
$a_4 = a_3 + d = a_1 + 3d = 1 + 3 \times 4 = 13$
…
$\boldsymbol{a_n = a_1 + (n-1)d}$

So $a_{100} = a_1 + 99d = 1 + 99 \times 4 = 397$

2.Property of an arithmetic sequence

A property of an arithmetic sequence is that any term is the arithmetic mean of its neighbors.

$$a_n = \frac{a_{n-1} + a_{n+1}}{2}$$

Proof:

$$a_n = a_{n-1} + d$$
$$a_n = a_{n+1} - d$$

Add the left and the right sides:

$$2a_n = (a_{n-1} + d) + (a_{n-1} - d)$$
$$2a_n = a_{n-1} + a_{n-1}$$

Dividing by 2:

$$a_n = \frac{a_{n-1} + a_{n-1}}{2}$$

Another property of arithmetic sequences is that we can find the common difference d if we know any 2 terms a_s and a_t

$$d = \frac{a_s - a_t}{s - t}$$

3. Sum of an arithmetic sequence

$$S = a_1 + a_2 + a_3 + \cdots + a_n = n \times \frac{a_1 + a_n}{2}$$

Proof: we write the sum in 2 ways, in increasing order and in decreasing order:

$S = a_1 + a_2 + a_3 + \cdots + a_n$
$S = a_n + a_{n-1} + a_{n-2} + \cdots + a_1$

Adding up left and right sides:

$$2S = (a_1 + a_n) + (a_2 + a_{n-1}) + (a_3 + a_{n-2}) + \cdots$$

We notice that:

$$a_1 + a_n = a_2 + a_{n-1} = a_3 + a_{n-2} = \cdots$$

$$2S = (a_1 + a_n) \times n$$

$$S = \frac{(a_1 + a_n) \times n}{2}$$

4. Arithmetic sequences -summary

$$a_n = a_1 + (n-1)d$$
$$a_n = \frac{a_{n-1} + a_{n+1}}{2}$$
$$d = \frac{a_s - a_t}{s - t}$$
$$S = \frac{(a_1 + a_n) \times n}{2}$$

Homework problems are on the next page: 

Homework problems

Instructions: Please always write solutions on a ***separate sheet of paper***. Solutions should include explanations. I want to see more than just an answer: I also want to see how you arrived at this answer, and some justification why this is indeed the answer. So **please include sufficient explanations**, which should be clearly written so that I can read them and follow your arguments.

1. Write the first 5 terms of an arithmetic sequence if $a_1 = 7$ and $d = 2$.

2. What are the first two terms for the sequence $a_1, a_2, -9, -2, 5, \ldots$?

3. $a_{10} = 131$ and $d = 12$. What is a_1?

4. $a_5 = 27$ and $a_{27} = 60$. Find the first term a_1 and the common difference d.

5. Find the common difference d in an arithmetic sequence if the 9-th term is 18 and the 11-th term is 44.

6. In the arithmetic progression 5, 17, 29, 41, . . . what term has a value of 497?

7. Find the sum of the first 10 terms for the series: 4, 7, 10, 13, . . .

8. Find the sum of the first 1000 odd numbers.

9. Find the sum 2 + 4 + · · · + 2018.

10. In a given arithmetic progression, the first term is 6, and the 87-th term is 178. Find the common difference of this arithmetic progression and give the value of the first five terms.

11. The 3-rd term of the arithmeti progression is equal to 1. The 10-th term of it is three times as much as the 6-th term. Find the first term and the common difference. (**Hint:** Use the formula for the n-th term of the progression and write what is given in the problem using this formula.)