Homework 2: Algebraic identities and factorization.

HW is Due Oct 6th

Basic algebraic identities for refreshing your memory:

Exponents Laws

If a *and* b are real numbers and n is a positive integer

$(ab)^n = a^n b^n$ (eq. 1)

$\sqrt{ab} = \sqrt{a}\sqrt{b}$ (eq. 2)

$(a+b)^2 = a^2 + 2ab + b^2$ (eq.3)

$(a-b)^2 = a^2 - 2ab + b^2$ (eq.4)

And also: $a^2 - b^2 = (a-b)(a+b)$ (eq. 5)

And:

$$a^3 + b^3 = (a+b)\,(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a-b)\,(a^2 + ab + b^2)$$

Homework problems

Instructions: Please always write solutions on a ***separate sheet of paper***. Solutions should include explanations. I want to see more than just an answer: I also want to see how you arrived at this answer, and some justification why this is indeed the answer. So **please include sufficient explanations**, which should be clearly written so that I can read them and follow your arguments.

1. Using algebraic identities calculate
   a. $299^2 + 598 + 1$
   b. 199^2
   c. $51^2 - 102 + 1$
   d. $16^3 - 15^3$
   e. $57^3 - 56^3$

2. Expand
   a. $(4a - b + c)^2 =$
   b. $(a + 9)(a - 9) =$
   c. $(3a - 2b)^2 =$

3. Factorize (i.e., write as a product) the following expressions:
   a. $p^2 - 2pq + q^2$
   b. $9x^{2a} - 12x^a + 4$
   c. $x(z-a)^2 - y(a-z)$
   d. $x^3 + x^2 + x + 1$
   e. $(a+b+c)^2 + ax + bx + cx$
   f. $1-y^2$
   g. $x^4 - y^4$
   h. $m^2 - 9p^2q^2$
   i. $a^4 - 169b^4$
   j. $(x+y)^2 - (x-y)^2$
   k. $a^2 + 3a + 2$
   l. $x^2 + 5x + 4$
   m. $x^2 - x - 12$
   n. $x^2 - x - 2$
   o. $(2x-y)^{n+1} - (2x-y)^n$