

MATH 6
HANDOUT 18: COLORING AND CUTTING

In problems below it is useful to color the object – maybe as a chessboard, or in some other way. Here are a couple of problems we solved in class.

- Imagine that we cut a lower left and upper right cells of a chessboard. Can we tile it with dominoes 2×2 ?

Solution: Each domino covers black and white cells, so for the board to be coverable with dominoes, the number of black and white squares should be the same. But in this problem, we cut out cells of the same color, so the answer is no.

- A knight starts in the lower left corner of the chessboard and in a few turns ends up in the upper right corner. Is the number of steps it made odd or even?

Solution: Even: the color of the square changes with each turn, and opposite corners have the same color.

- Numbers are placed in squares of a 3×3 board, as shown below. Each turn, you may either add or subtract the same number from two neighboring cells. Can you get the square on the right?

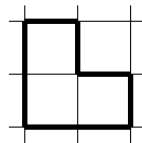
2	6	2
4	7	3
3	6	5

1	0	0
0	2	0
0	0	1

Solution: Use the checkerboard pattern. Initially, sum of white numbers is equal to the sum of black numbers, and it doesn't change – which is not the case for the square on the right.

HOMEWORK

- In each square of a 2011×2011 board there is a bug. At some moment each bug jumps into one of the adjacent squares. Explain why after that one of the squares will be empty. [Hint: how many bugs there were on black squares? on white squares?]
- Can one color a quad ruled paper using 3 colors so that each 1×3 rectangle contains squares of each of 3 colors?
- Kathryn had a construction kit which has 12 1×3 wooden tiles. Another student from our class has stolen one of the tiles and replaced it with an L-shaped one. Can Kathryn make a 6×6 square using these tiles?



- Can you cut an 8×8 board into 15 horizontal and 17 vertical 1×2 tiles?
- You have a $3 \times 3 \times 3$ cube with the central $1 \times 1 \times 1$ small cube removed. Is it possible to cut it into $1 \times 1 \times 2$ pieces?
- A piece of cheese has the shape of a $3 \times 3 \times 3$ cube with the central $1 \times 1 \times 1$ small cube removed. A mouse starts eating the cheese eating one $1 \times 1 \times 1$ cube at a time and then moving to a next one (so that the next one has a common face with the one it had just eaten). Can the mouse eat all the cheese?

7. In a computer game, a wizard is more powerful than an orc, so when a wizard fights an orc, he has 60% chance of winning. If a wizard fights one by one a group of 5 orcs, what are the chances that he will defeat them all?
8. In how many ways can one arrange 5 books on a shelf?
- *9. In how many ways can you arrange 5 books on 2 shelves? Order on each shelf matters.