MATH 6: HANDOUT 16
COORDINATES I

Coordinate Geometry: Introduction

In this section of the course we are going to study coordinate geometry. The basic notion is the **coordinate plane** – a plane with a given fixed point, called the **origin**, as well as two perpendicular lines – **axes**, called the x-**axis** and the y-**axis**. x-axis is usually drawn horizontally, and y-axis — vertically. These two axes have a **scale** – "distance" from the origin.

The scales on the axes allow us to describe any point on the plane by its **coordinates**. To find coordinates of a point P, draw lines through P perpendicular to the x- and y-axes. These lines intersect the axes in points with coordinates x_0 and y_0. Then the point P has x-coordinate x_0, and y-coordinate y_0, and the notation for that is: $P(x_0, y_0)$.

The **midpoint** M of a segment AB with endpoints $A(x_1, y_1)$ and $B(x_2, y_2)$ has coordinates:

$$M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

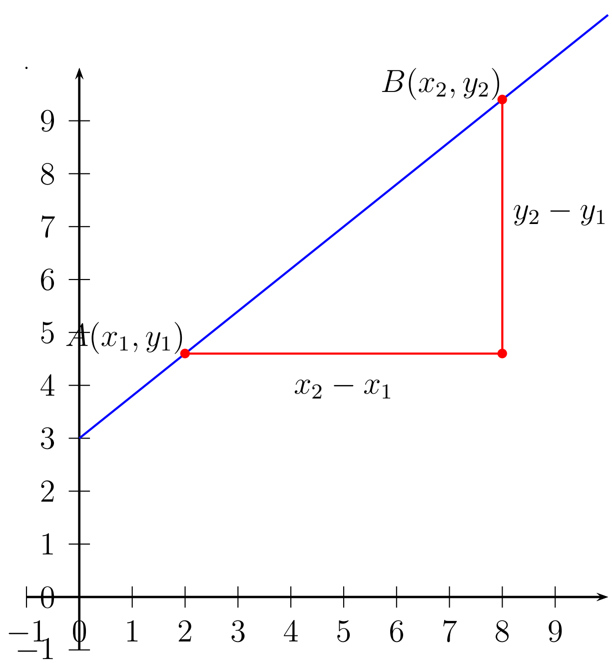
Lines

Given some relation which involves variables x, y (such as $x + 2y = 0$ or $y = x^2 + 1$), we can plot on the coordinate plane all points $M(x, y)$ whose coordinates satisfy this equation. Of course, there will be infinitely many such points; however, they usually fill some smooth line or curve. This curve is called the **graph** of the given relation.

Every relation (**equation**) of the form:

$$y = mx + b$$

where m, b are some numbers, defines a straight line. The slope of this line is determined by m: as you move along the line, y changes m times as fast as x, so if you increase x by 1, then y will increase by m:



In other words, given two points $A(x_1, y_1)$ and $B(x_2, y_2)$ **slope** can be computed by dividing change of y: $y_2 - y_1$ by the change of x: $x_2 - x_1$:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Two non vertical lines are **parallel** if and only if they have the **same slope**.

In the equation $y = mx + b$, b is a y**-intercept**, and determines where the line intersects the vertical axis (y-axis).

The equation of the **vertical** line is $x = k$, and the equation of the **horizontal** line is $y = k$. Notice that in case of the vertical line, the slope is undefined.

HOMEWORK

1. A point B is 5 units above and 2 units to the left of point $A(7, 5)$. What are the coordinates of point B?

2. Find the coordinates of the midpoint of the segment AB, where $A = (3, 11)$, $B = (7, 5)$.

3. Draw points $A(4, 1)$, $B(3, 5)$, $C(-1, 4)$. If you did everything correctly, you will get 3 vertices of a square. What are coordinates of the fourth vertex? What is the area of this square?

4. (a) 3 points $A(0, 0)$, $B(1, 3)$, $D(5, -2)$ are vertices of a parallelogram $ABCD$. What are the coordinates of point C?

(b) 3 points $A(0, 0)$, $B(2, 3)$, $D(4, 1)$ are vertices of a parallelogram $ABCD$. What are the coordinates of point C?

(c) 3 points $A(0, 0)$, $B(1, 5)$, $D(3, -2)$ are vertices of a parallelogram ABCD. What are the coordinates of point C?

(d) Can you guess the general rule: if $A(0, 0)$, $B(b_1, b_2)$, $D(d_1, d_2)$ are 3 vertices of a parallelogram, what are coordinates of point C?

5. Consider the triangle ΔABC with the vertices $A(-2, -1)$, $B(2, 0)$, $C(2, 1)$. Find the coordinates of the midpoint of B and C. Find the length of the median (i.e. a median unites a vertex with the midpoint of the opposite side) from A in the triangle ΔABC.

6. What is the slope of a line whose equation is $y = 2x$? What is the slope of a line whose equation is $y = mx$?

7. In this problem you will find equations that describe some lines.

(a) What is the equation whose graph is the y-axis?

(b) What is the equation of a line whose points all lie 5 units above the x-axis?

(c) Is the graph of $y = x$ a line? Draw it.

(d) Find the equation of a line that contains the points $(1, -1)$, $(2, -2)$, and $(3, -3)$.

8. For each of the equations below, draw the graph, then draw the perpendicular line (going through the point (0, 0)) and then write the equation of the perpendicular line

(a) $y = 2x$ (b) $y = 3x$

(c) $y = -x$ (d) $y = -\frac{1}{2}x$

Can you determine the general rule: if the slope of a line is k, what is the slope of the perpendicular line?