Math 5e, Fall 2024 Homework 15

due January 22

Instructions: Some of the problems we solved in class, and some are new. Please try to solve all problems, do your best, and show your work. Write on separate sheets of paper, not between the lines of this handout!

Binary Numbers: Numbers represented by using only 0s and 1s.

Powers of 2

n	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*
2^n	***1***	***2***	***4***	***8***	***16***	***32***	***64***	***128***	***256***	***512***

Numbers in decimal notation can be presented like this (same as converting a number to a decimal notation): $351 = 3 \times 100 + 5 \times 10 + 1 \times 1$

- Similarly, to convert a number into a binary, we need to represent it in powers of 2:
  $351 = \mathbf{256} + 95 = \mathbf{256 + 64} + 31 = \mathbf{256 + 64 + 16} + 15 = \mathbf{256 + 64 + 16 + 8} + 7 = \mathbf{256 + 64 + 16 + 8 + 4 + 2 + 1}$
  $351 = 1\times2^8 + 0\times2^7 + 1\times2^6 + 0\times2^5 + 1\times2^4 + 1\times2^3 + 1\times2^2 + 1\times2^1 + 1\times2^0 = 101011111$
- Or use the division by 2 method from class and write the remainders in reverse order.

- To convert numbers from binary to decimal, we use the familiar rule to multiply each digit by the position value in base 2. For example:
  $1010 = 1\times2^3 + 0\times2^2 + 1\times2^1 + 0\times2^0 = 8 + 0 + 2 + 0 = 10$

Product formula: $(x - a)(x + a) = (x^2 - a^2)$

Homework problems

1. Solve the following equations:
   (a) $2(x - 1) = \frac{2}{3}(x + 5)$

   (b) $2x - 5(x - 7) = -1$

   (c) $\frac{x-2}{x-5} = 3$

2. Do the following arithmetic operations with binary numbers.

   (a) 110101 + 111011

   (b) 10101 × 1011

   (c) (10101 + 1101) × 10110

3. Your teacher ordered a birthday cake with her age in binary as 110010. What is her age if decimals? In Roman numerals?

4. What is the largest number that can be written as a 6-digit binary number? (Hint: what is the smallest 7-digit binary number?)

5. (Started in class) The following is the beginning of a computer file. Can you decode it (assuming it is written in the standard Latin 1, encoding)? **Use the encoding table from the Class Notes.**

   01010100 01101111 01110000 00100000 01110010 01100101 01100011 01110010 01100101 01110100 00001010

6. Decode "I study math at School Nova." in Standard Latin -1 binary. Use the encoding table to create a binary string of 8-bit character codes.

7. (Optional) In order to allow computers to deal with different languages, computer scientists have developed so-called Unicode, a standard list of symbols covering virtually all human languages, from Armenian to Vietnamese. In particular, it includes Latin letters, Cyrillic letters, Chinese characters (hanzi), Emoji, and more.

   The latest revision of Unicode (v16) contains about 154,998 symbols. If we want to represent each of them by a sequence of 0 and 1, would it be enough to use 16 bits (0s and 1s) for each symbol? If not, what is the smallest number of bits per symbol one would need?
   Hint: what is 16 -bits in powers of 2? Use that $2^{10} \sim 1000$