MATH 5e: Class Work 15

Topics: Operations with binary numbers

- Negative powers and powers in the denominator: $\frac{a^n}{a^m} = a^{n-m}$; $a^n = \frac{1}{a^{-n}}$; and $a^{-n} = \frac{1}{a^n}$
- The formula for fast multiplication: $(x-y)(x+y) = x^2 - y^2$
- Binary numbers: Numbers represented by using only 0s and 1s
- **Covert from a binary to a 10-base number:**
  Write down the binary number and list the powers of 2 from right to left under each digit, starting with power 0 and increasing to the left. Multiply each digit of the binary number by 2 raised to the power of its position. Then, add all the values together to get the decimal equivalent.

Rules: Covert from a 10-base number to a binary

Write the number and find the closest power of 2 that is less than the number.
Subtract the original number and the closest power – use the difference. Find the nearest power of two smaller than the difference, repeat until you get 0 or 1.
List all powers from left to right, starting with the largest you have to 2^0. Under each power of 2, write 1 if you have it and 0 if you do not.

Example: $22 = \mathbf{16} + 6 = \mathbf{2^4} + \mathbf{6} = 2^4 + \mathbf{4} + 2 = 2^4 + \mathbf{2^2} + 2 = \mathbf{2^4} + \mathbf{2^2} + 2^1 + 0$

$22 = 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 = 10110$

Division (easier) method

- Divide the 10-base number by 2
- Write down the remainder, which will be either 0 or 1
- Repeat step 1 with the quotient until you reach 0
- Write down the remainders in reverse order to get the binary number

Example: $22:2 = 11 \ , R\,0$

$11:2 = 5 \ , R\,1$

$5:2 = 2 \ , R\,1$

$2:2 = 1 \ , R\,0$

- $1:2 = 0 \ , R\,1 \quad 22 = 10110$

Do on your own

1. $3^4 =$ $3 \times 10^4 =$
2. Write in scientific notation 12 5300 052 =
3. Calculate as power $\frac{3^6}{3^7} =$ and $\frac{3^6}{3^{-7}} =$

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Problems

Review: Fill in the table with powers of 2 (as many as you can)
(2^n has base is 2, and the exponent n = 0, 1, 2 ….)

$\boldsymbol{n}$	$n = 0$	1	2	3	4	5	6	7	8	9	10
$\boldsymbol{2^n}$	$2^0 = 1$										

Review: from HW 14, what is one ly and 1 parsec?

1. Convert to decimals

   a) 10, 100, 10000 … then

   b) $101 = 100 + 1 =$

   c) 1001

   d) 11110

   e) 11011

   f) 100100

2. Convert to binary – use the closest powers of 2 method

   a) 9

   b) 12

   c) 24

   d) 38

   e) 45

3. Arithmetic operations in binaries. Perform in binaries, then convert and check the result in decimals

   a) Add
   $10110 + 1101$

   b) Multiply. Try to do it alone (Hint: how do you multiply by 1, or by 0?)
   1010×11

   $11011\ x\ 101$

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4. Your teacher ordered a birthday cake with her age in binary as 110010. What is her age if decimals? In Roman numerals?

5. What is the largest number that can be written as a 6-digit binary number? (Hint: what is the smallest 7-digit binary number?)

6. A car travels 125 miles during some period of time. During the same period, another car that is 10 mph faster than the first has traveled 150 miles. What is the speed of the faster car?

If time

7. In computers, letters and other symbols are written as sequences of 0 and 1. A bit is the smallest unit of digital information and is short for "binary digit." Bits are the fundamental building blocks of all computing systems and digital communications
    a) How many bits are needed to code all 26 letters of the English alphabet in binary (a binary number that can hold 26 different combinations)?
    b) If we want to have lower and upper case letters, punctuation, and numbers – we need more, so usually 8 bits are used per symbol. 8 bits = 1 byte. How many combinations can we store in 1 byte?
    c) The table that has the Latin letters and their binary codes is called an encoding table (Latin 1, ISO 8859-1). What is the lowercase **a** in binary?

8. The latest computers use 32 or 62 bits to store information.
    a) How many symbols/letters/emojis can we store in 32 bits using 8-bit encoding?
    b) If we use all 32 bits for one symbol, how many symbols can we have in a table?

9. The following is the beginning of a computer file. Can you decode it (assuming it is written in the standard Latin 1 encoding)?

    01010100 01101111 01110000 00100000 01110011 01100101 01100011 01110010 01100101 01110100 00001010.