Math 5e, Fall 2024 Homework 14

Homework #14 is due January

Instructions: Some of the problems we solved in class, and some are new. Please try to solve all problems, do your best, and show your work. Write on separate sheets of paper, not between the lines of this handout!

Binary Numbers: Numbers represented by using only 0s and 1s.

Powers of 2

n	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*
2^n	***1***	***2***	***4***	***8***	***16***	***32***	***64***	***128***	***256***	***512***

Numbers in decimal notation can be presented like this (same as converting a number to a decimal notation): $351 = 3 \times 100 + 5 \times 10 + 1 \times 1$

- Similarly, to convert a number into a binary, we need to represent it in powers of 2:
  $351 = \mathbf{256} + 95 = \mathbf{256 + 64} + 31 = \mathbf{256 + 64 + 16} + 15 = \mathbf{256 + 64 + 16 + 8} + 7 = \mathbf{256 + 64 + 16 + 8 + 4 + 2 + 1}$
  $351 = 1\times2^8 + 0\times2^7 + 1\times2^6 + 0\times2^5 + 1\times2^4 + 1\times2^3 + 1\times2^2 + 1\times2^1 + 1\times2^0 = 101011111$
- Or use the division by 2 method from class and write the remainders in reverse order.

- To convert numbers from binary to decimal, we use the familiar rule to multiply each digit by the position value in base 2. For example:
  $1010 = 1\times2^3 + 0\times2^2 + 1\times2^1 + 0\times2^0 = 8 + 0 + 2 + 0 = 10$

Product formula: $(x - a)(x + a) = (x^2 - a^2)$

Homework problems

1. Solve the following equations:
   a) $3(x - 1) - 4 = 3x + 8$
   b) $\frac{1}{2}(x - 1) = -19$
   c) $|2x| = 10$

2. Convert the decimal numbers to binary:
   9, 12, 24, 38, 45

3. Convert the following binary numbers to decimal:
   101, 1001, 10110, 11011, 10101

5. Can you find whole numbers a; b such that $a^2 - b^2 = 17$? [Hint: Use the formula we talked about in class, and think about what $a - b$ and $a + b$ must be.]

4. Factor the following number into primes: $99^2 - 9^2$. [Hint: you do not have to compute this number, just the two factors as powers]

6. For the following problem, you need to know that the speed of light is about 300,000 km/sec, and one year is about 3×10^7 seconds. Answer using scientific notation (or a number $\times$ power of 10).

a) In astronomy, a common unit of distance is a light year: the distance light covers in one year. How many kilometers is it?
b) Another common unit of distance in astronomy is a parsec, which is approximately equal to 3×10^{13} km.
   - Can you compute how many parsecs are there in one light year?
   - How many parsecs is the distance between Earth and Sun (The distance is about 1.5×10^8km)?
   - How many parsecs is the distance between Earth and the Andromeda Nebula (2,000,000,000,000,000,000,000 km)?