MATH 5e: Class Work 14

Topics: Practice powers, introduction to binary numbers

- Negative powers and powers in the denominator: $\frac{a^n}{a^m} = a^{n-m}$

$$a^n = \frac{1}{a^{-n}} \quad \text{and} \quad a^{-n} = \frac{1}{a^n}$$

- The formula for fast multiplication: $(x-y)(x+y) = x^2 - y^2$
- Binary numbers: Numbers represented by using only 0s and 1s

P r o b l e m s

1. Calculate using the rules for powers, and pay attention to powers in the denominator.

We have the following variables: $x = a^3 \cdot b^2 \quad y = \frac{b^5}{a^2 c^4} \quad z = \frac{c^3}{ab}$

a) $xyz =$

b) $(xy)^2 z =$

c) $\frac{x}{y} =$

d) $\frac{x}{y^{-1}} =$

2. Open the parenthesis and simplify the expressions.
   a) $(2-x)(2+x) =$
   b) $y(y-6) - 6 =$
   c) $(3x+y)(3x-y) =$
   d) $\frac{x^2-y^2}{3(x+y)} =$

What can be concluded about expression in the form: $a^2 - b^2 =$

3. Fill in the table with powers of 2 (2^n has base is 2, and the exponent n = 0, 1, 2 ….)

$\boldsymbol{n}$	$n = 0$	1	2	3	4	5	6	7	8	9	10
$\boldsymbol{2^n}$	$2^0 = 1$										

4. Binary numbers are written using only two digits – 0 and 1. To count and then perform simple operations with binary numbers, such as addition and subtraction, we use place values corresponding to powers of 2.

Base-10 numbers	Base-2 (binary) numbers
0	0
1	1
2	
3	
4	
5	
6	
7	
8	
Place values	2^3 2^2 2^1 2^0

Rules: Covert from a binary to a 10-base number:

Write down the binary number and list the powers of 2 from right to left under each digit, starting with power 0 and increasing to the left. Multiply each digit of the binary number by 2 raised to the power of its position. Then, add all the values together to get the decimal equivalent.

5. Convert from binary to decimal

   a) 10110

   b) 110

   c) 11

   d) 10110

6. Add the binary numbers.

   a) $1100 + 111 =$
   b) $101 + 10001 =$

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Rules: Covert from a 10-base number to a binary

- Write the number and find the closest power of 2 that is less than the number.
- Subtract the original number and the closest power – use the difference. Find the nearest power of two smaller than the difference, repeat until you get 0 or 1.
- List all powers from left to right, starting with the largest you have to 2^0. Under each power of 2, write 1 if you have it and 0 if you do not.

Example: $22 = \mathbf{16} + 6 = \mathbf{2^4} + \mathbf{6} = 2^4 + \mathbf{4} + 2 = 2^4 + \mathbf{2^2} + 2 = \mathbf{2^4} + \mathbf{2^2} + 2^1 + 0$

$22 = 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 = 10110$

Division (easier) method

- Divide the 10-base number by 2
- Write down the remainder, which will be either 0 or 1
- Repeat step 1 with the quotient until you reach 0
- Write down the remainders in reverse order to get the binary number

Example: $22 : 2 = 11 \ , R\ 0$

$11 : 2 = 5 \ , R\ 1$

$5 : 2 = 2 \ , R\ 1$

$2 : 2 = 1 \ , R\ 0$

$1 : 2 = 0 \ , R\ 1 \quad 22 = 10110$

7. Convert to binary:

   a) 11

   b) 32

   c) 6

   d) 351

8. What is your age in binary?

9. What is the largest number that can be written as a 5-digit binary number? (Hint: what is the smallest 6-digit binary number?)

If time

10. A car travels 125 miles during some period of time. During the same period, another car that is 10 mph faster than the first has traveled 150 miles. What is the speed of the faster car?