Math 5e, Fall 2024 Homework 12

Homework #12 is due December 18

Instructions: Some of the problems we solved in class, and some are new. Please try to solve all problems, do your best, and show your work. Write on separate sheets of paper, not between the lines of this handout!

General notation (n is a whole number):

$$a^n = a \times a \times a \times \ldots \times a \ (n \text{ times})$$

Special cases:

$a^0 = 1$	read: a-to-the-zero
$a^1 = a$	is just itself 'a'
$a^2 = a \times a$	read: a-squared
$a^3 = a \times a \times a$	read: a-cubed

Product $a^n a^m = a^{n+m}$

Division $\frac{a^n}{a^m} = a^{n-m}$

$a^n = \frac{1}{a^{-n}}$ and $a^{-n} = \frac{1}{a^n}$

Power of a product $(ab)^n = a^n \times b^n$

Power raised to a power $(a^m)^n = a^{m \times n}$

In base-4 number system we have only 4 digits: 0, 1, 2, 3.
When we need to count 4 items, we do not have digit 4 and cannot use it. We have to use the next place value position (similarly to moving from the place value for the once to the place value for the tens in the Base -10 number system). Let's name 4 items with a □, and 16 items with 4 × □.
Then a number can be presented as a sum of powers of 4.
Example: $10 = 2 \times 4^1 + 2 \times 4^0$

1. Simplify the following and show the answer in the exponent (power) form

(a) $\frac{3^7 \cdot 2^7}{2^3 \cdot 2^4} =$

(b) $\frac{6^5 \cdot 2^4}{3^5 \cdot 2^2} =$

(c) $\frac{7^9 \cdot 2^5}{7^2 \cdot 2^4} =$

(d) $\frac{11^4}{11^2 \cdot 5^2 \cdot 5^3} =$

(e) $7^4 \cdot 11^2 \cdot 11^{-5} \cdot 7^2 =$

(f) $\frac{3^{-5} \cdot 2^7}{3^{-3} \cdot 2^4} =$

(g) $\frac{42^2}{6^2} =$

(h) $\frac{3^5 \cdot 3^{-5}}{3^9} =$

2. Solve the equations:

(a) $\frac{9}{13}z = 3$ (b) $|x| = 2$ (c) $|x + 11| = 2$

(d) $\frac{x}{2} + 1 = \frac{4x}{7}$ (e) $x = \frac{1}{4}x + 6$ (f) $\frac{x+3}{x+1} = 4$

Hint for (f): multiply both sides of the equation by ($x + 1$) to "remove" the denominator. Be careful, as you cannot divide by zero, your answer should not be x = -1

3. Compute, but be very attentive to signs and the order of operations (first: operations in brackets, then multiplication or addition, then addition or subtraction). Show all the steps!

(a) $(-5 - 9) \div (-2) + 7 =$

(b) $-2(-5 - 9) - 7 \times 4 =$

(c) $-9 + 14 \div (-2) + 7 =$

(d) $(-2) \times (-2) \times (-2) \times (-2) \times (-2) =$

(e) $-16 \div (-8) =$

(f) $-16 \div 8 =$

(g) $16 \div (-8) =$

4. Anna has 60 coins which should be identical but one of them is fake. The fake one looks the same as all other coins but is lighter. Using balance scales but no weights and placing coins on both platforms, what is the fastest way to find the fake coin? What would you do if you did not know whether the fake coin was lighter or heavier than the real ones?

5. A fish head weighs as much as the tail and half of the body together. The body weighs as much as the head and tail together. If the tail weighs 1 kg, how heavy is the fish?

6. Base 4 numbers:
   a) add two base-4 numbers together:

123	3201
+ 321	+ 2310

[Do not add in base-10, these are base-4 numbers.]

b) Write a formula, instruction, or algorithm on how to translate base-4 number ***abcd*** to base-10 number, where ***a***, ***b***, ***c***, ***d*** can be 0, 1, 2, or 3.

c) Translate the numbers and the results from a) into the base-10 system

7. (*) Here are phrases in Swahili with their English translations:
   atakupenda – He will love you.
   nitawapiga – I will beat them.
   atatupenda – He will love us.
   anakupiga – He beats you.
   nitampenda – I will love him.
   unawasumbua – You annoy them.

Translate the following into Swahili:

You will love them.
I annoy him.