MATH 5e: Class Work 12

Topics: More on powers. Base 10 and base 4 systems

- Powers

General notation (*n* is a whole number):

$a^n = a \times a \times a \times ... \times a$ (*n* times)

Special cases:

$a^0 = 1$	read: *a*-to-the-zero
$a^1 = a$	is just itself '*a*'
$a^2 = a \times a$	read: *a*-squared
$a^3 = a \times a \times a$	read: *a*-cubed

Product $a^n a^m = a^{n+m}$

Division $\frac{a^n}{a^m} = a^{n-m}$

$a^n = \frac{1}{a^{-n}}$ and $a^{-n} = \frac{1}{a^n}$

Power of a product $(ab)^n = a^n \times b^n$

Power raised to a power $(a^m)^n = a^{m \times n}$

- Base-4 system

In base-4 number system we have only 4 digits: 0, 1, 2, 3.

When we need to count 4 items, we do not have digit 4 and cannot use it. We have to use the next place value position (similarly to moving from the place value for the once to the place value for the tens in the base -10 number system). Let's name 4 items with a □, and 16 items with 4 × □.

Then a number can be presented as a sum of powers of 4.

Example: $10 = 2 \times 4^1 + 2 \times 4^0$

Problems

1. **Review:** equations with absolute values
   a) $|x| = 5$
   b) $|y - 8| = 12$
   c) $z + |-6| = -15$
   d) $|y| = -3$

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2. Compare base-10 to base-4 systems
   Base-10 uses 10 different digits to write numbers, and positions (place value)

   0
   1
   2
   3
   4
   5
   6
   7
   8
   9

   After the digits are used, we use 0 again and place 1 in front – meaning 10

Similarly, in base-4 – we can use only 4 digits: 0, 1, 2, 3. What do we do if we need 4?

3. Present the numbers in base-4 using powers of 4
   a) $10_{10} =$
   b) $25_{10} =$
   c) $65_{10} =$

4. Write the numbers in base-10 and base-4 as powers of 10 and powers of 4

   $222_{10} =$

   $222_4 =$

5. Perform the addition in base-4. Then convert the numbers in base-10 and check your answers
   a) $23_4 + 11_4$
   b) $333_4 + 21_4$

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Operations with Powers

6. Operations with powers (Review)

   a) $\frac{a^2}{a^3}$

   b) $a^2 \cdot a^3 =$

   c) $(a^2)^4 =$

   d) $(a^2b)^4 =$

   Homework Problem 2 (Review)

   a) $(2z^2.3z^3.z)^2$

   b) $(4c^2.c^3)^3$

   c) $\left(\frac{5\,g^4b^5}{4\,g^2b^3}\right)^3$

   d) $\left(\frac{8dg^2}{3d^3g^4}\right)^3$

7. Find the unknown power

   $(3^2)^m = 3^{10}$ $(7^n)^4 = 7^{12}$ $5^3 \cdot x = 5^7$ $3^x = 27$

8. Simplify and present as a product of powers (on your own)

   a) $\left(\frac{9\,a^7b^6}{45\,a^3b}\right)^4$

   b) $2x^2 \cdot x^3 - x^7 \div x^2 =$

   c) $((x^2y)^3)^4 =$

   d) $\frac{(-ab)^8}{(ab)^2} =$

If time

9. Word problem: create a system of equations.

   Anushka and Vivian have 93 dollars from working over the summer. Vivian and Isabella have 104 together, and Isabella and Anushka have 95 dollars together.