

MATH 5e: Class Work 11

Topics: More on powers. Arithmetic operations

- Powers notation

General notation (n is a whole number):

$$a^n = a \times a \times a \times \dots \times a \text{ (} n \text{ times)}. \quad a \text{ is called the base, and } n \text{ – the exponent}$$

Special cases:

$$a^0 = 1 \quad \text{read: } a\text{-to-the-zero}$$

$$a^1 = a \quad \text{is just itself 'a'}$$

$$a^2 = a \times a \quad \text{read: } a\text{-squared}$$

$$a^3 = a \times a \times a \quad \text{read: } a\text{-cubed}$$

- Multiplication and division of powers with the same base

$$a^n a^m = a^{n+m}$$

Because:

$$a^n a^m = \underbrace{(a \times a \times a \dots)}_{n\text{-times}} \times \underbrace{(a \times a \times a \dots)}_{m\text{-times}} = \underbrace{a \times a \times a \times a \dots}_{n+m \text{ times}}$$

$$a^n : a^m = \frac{a^n}{a^m} = a^{n-m} \quad \text{if } n > m \quad \text{or} \quad \frac{a^n}{a^m} = \frac{1}{a^{m-n}} \quad \text{if } m > n$$

$$a^n = \frac{1}{a^{-n}}$$

$$a^{-n} = \frac{1}{a^n}$$

Then we define $\frac{a^n}{a^n} = a^{n-n} = a^0 = 1$

- Power of a product

$$(ab)^n = a^n \times b^n$$

because

$$(ab)^n = ab \times ab \times ab \times \dots \times ab \text{ (} n \text{ times)} = (a \times a \times a \times \dots \times a) \times (b \times b \times b \times \dots \times b) \text{ (} n \text{ times)}$$

- Power raised on a power

$$(a^m)^n = a^{m \times n}$$

example

$$(a^2)^3 = a^2 \times a^2 \times a^2 \text{ (3 times)} = a \times a \times a \times a \times a \times a \text{ (6 times)} = a^6$$

MATH 5e: Class Work 11

Problems

1. **Review:** HW problems
2. List powers of 2^n and 3^n and for $n = 1$ to 8
3. Express with powers of 10, multiply and state the answer in powers of 10, with a prefix, in scientific notation, the following number

$$120\,000 \times 300\,000\,000 =$$

4. Calculate in the most efficient way.

a) $5^{12} \cdot 5^{-11} =$

b) $5^3 \cdot \frac{5^3}{5^5} =$

c) $(-5)^2 \cdot (-2)^2 =$

d) $(-12)^3 \cdot 4^{-3} =$

5. Present as a power.

a) $(x^2)^3$; $(2^3)^4$; $\left(\left(\frac{1}{2}\right)^2\right)^3$

b) $\left(\frac{5^2 \times 5^4}{5^3 \times 5^2}\right)^2$; $\left(\frac{10^0 \times 10^{10}}{5^8 \times 10^2}\right)^3$

c) $x^5 \cdot (x^2)^3$; $(x^3)^4 : x^8$

6. Represent the numbers with one but smaller base.

a) 25^4

b) $\frac{64^4}{16^4}$

c) $(3 \cdot 27)^3$

7. Simplify the following expressions.

a) $\frac{(-2)^7 - 2^4}{(-2)^4} =$

MATH 5e: Class Work 11

b) $\frac{2^8 \cdot 3^6}{2^4 \cdot 9^3} =$

c) $\frac{2^6 \cdot 9^2 \cdot 14^2}{4^2 \cdot 3^5 \cdot 7} =$

Arithmetic operations

8. Open the brackets and simplify the expressions

a) $(x - 5)(2x + 1) =$

b) $(x + 7)(x^2 - 2x) =$

c) $(2y^3 - 3y + 2)(y - 3) =$

d) $(x + 3)(x - 3) - x(x - 2) =$

9. Simplify the following expressions using the power rules (from HW)

a) $2^{-2}(2^2 + 4^2) =$

b) $6^3(2^{-3} + 3^{-3}) =$

If time

10. Find the unknown

a) $(3^2)^m = 3^{10}$

b) $(7^n)^4 = 7^{12}$

c) $5^3 \cdot x = 5^7$

d) $3^x = 27$