Math 5e, Fall 2024 Homework 10

Homework #10 is due December 4

Instructions: Some of the problems we solved in class, and some are new. Please try to solve all problems, do your best, and show your work. Write on separate sheets of paper, not between the lines of this handout!

General notation (n is a whole number):

$$a^n = a \times a \times a \times \ldots \times a \ (n \text{ times})$$

Special cases:

$a^0 = 1$	read: a-to-the-zero
$a^1 = a$	is just itself 'a'
$a^2 = a \times a$	read: a-squared
$a^3 = a \times a \times a$	read: a-cubed

Product $a^n a^m = a^{n+m}$

Division $\frac{a^n}{a^m} = a^{n-m}$

$a^n = \frac{1}{a^{-n}}$ and $a^{-n} = \frac{1}{a^n}$

Power of a product $(ab)^n = a^n \times b^n$

1. Solve the equations
   (a) $5 - x = -4 - 2x$

   (b) $7 - 2(1 - x) = -5$

   (c) $\frac{(x-2)}{(x-1)} = 3$

2. If you take half my age and add 7, you get my age 13 years ago. How old am I?

3. Simplify:

   (a) $\frac{(x^2y^2)x^3}{x^2y^5}$

   (b) $(3y^3 \cdot y^5)^2$

4. Let $a = 2 \cdot 10^8$, $b = 10^5$. Compute $a^2 \cdot b$, $\frac{a}{b}$, $a^2 \div b^3$. Remember $(a)^2 = (a) \times (a)$

5. How many cubic centimeters are there in one cubic kilometer = (1 km)^3? $(1km = 1000\ m, 1m = 100\ cm)$. Use powers of 10!

6. It is known that $2^{10} = 1024$, which is very close to 10^3. Use this to estimate the value of 2^{20}, and then 2^{32}.

Hint: $2^{20} = 2^{10+10} = 2^{10} \cdot 2^{10}$

7. Evaluate:
   (a) $(x-5)(2x+1) =$
   (b) $(x+7)(x^2-2x) =$

8. Solve:
   (a) $2^{-2} \cdot (2^2 + 4^2) =$
   (b) $6^3 \cdot (2^{-3} + 3^{-3}) =$
   (c) $\frac{2^8+2^{10}}{2^8} =$

9. The distance from Earth to the Sun is approximately 150 000 000 000 m. Write it as a product of two numbers with the largest possible power of 10.

10. Assume that a dandelion has 100 seeds. Each seed planted itself the next year and produced a dandelion with 100 seeds. If this continues year after year, how many dandelions will be there after 3 years? After how many years will we have 10 billion dandelions?