MATH 5e: Class Work 10

Topics: Multiplication and division of powers with the same base, power of a product.

- Powers notation

General notation (n is a whole number):

$a^n = a \times a \times a \times \ldots \times a$ (n times). a *is called the base*, and n *– the exponent*

Special cases:

$a^0 = 1$	read: a-to-the-zero
$a^1 = a$	is just itself 'a'
$a^2 = a \times a$	read: a-squared
$a^3 = a \times a \times a$	read: a-cubed

- Multiplication and division of powers with the same base

$$a^n a^m = a^{n+m}$$

Because: $a^n a^m = \underbrace{(a \times a \times a \ldots)}_{\text{n-times}} \times \underbrace{(a \times a \times a \ldots)}_{\text{m-times}} = \underbrace{a \times a \times a \times a \ldots..}_{\text{n+m times}}$

$a^n : a^m = \frac{a^n}{a^m} = a^{n-m}$ if $n > m$ or $\frac{a^n}{a^m} = \frac{1}{a^{m-n}}$ if $m > n$

$$a^n = \frac{1}{a^{-n}}$$

$$a^{-n} = \frac{1}{a^n}$$

Then we define $\frac{a^n}{a^n} = a^{n-n} = a^0 = 1$

- Power of a product

$$(ab)^n = a^n \times b^n$$

because

$(ab)^n = ab \times ab \times ab \times \ldots \times ab$ (n times) $= (a \times a \times a \times \ldots \times a) \times (b \times b \times b \times \ldots \times b)$ (n times)

Problems

1. **Review:** HW problems

2. Write the product as a power.
   a) $5^2 \cdot 5^3$
   b) $7^2 \cdot 7^5 \cdot 7$
   c) $a^2 \cdot a^4 \cdot a$
   d) $2^2 \cdot 64$
   e) 81×3^6

3. Find the prime factorization and write as a product of powers
   a) 225
   b) 72
   c) $720 \cdot 54$

4. Calculate using the rules for the product of powers.
   a) $2^3 \cdot (2 + 2^2)$
   b) $0.3^3 \cdot (0.3^2 - 0.3)$
   c) if $x + y = 4$ $\quad 2 \cdot 3^x \cdot 3^y$
   d) $(x + 2)(x - 2)$

Division

5. Calculate
   a) $2^5 : 2^3$
   b) $5^{-2} \cdot 5^3$
   c) $\frac{7^3}{7^5}$
   d) $\frac{3^{15}}{3^5.3^6}$
   e) $\frac{3^9.7^5}{7^6.3^8}$
   f) $2^{-3} \cdot (2 + 2^2)$

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Power of a product

6. The side of a square is a cm. By what factor will the area increase if the side is 3 times larger than before?

7. Represent the expression as a product of powers.
   a) $(5 \cdot 10)^2$
   b) $(3 \cdot 10)^3$
   c) $(5 \cdot 0.1)^2$

8. Calculate most efficiently.
   a) $2^4 \cdot 5^4$
   b) $0.5^{15} \cdot 2^{15}$

9. Powers of 10.
   a) Write the following numbers as powers of 10
      10 100 1000 10 000 100 000 1 000 000
   b) Using the powers of 10, write the numbers as a sum of powers of 10
      3 848
      62 017
   c) The speed of light is $3 \cdot 10^8$ $meters/second$. How many meters does the light pass in one second? How many kilometers in one second?

10. Many chemical elements are radioactive. This means that its quantity is reduced by half over some time period, called half-life. The radioactive element Francium (Fr) has a half-life of about 20 minutes. How much of the initial quantity will be left after 1 hour?

Powers of 10

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Prefix	Exponent	Number	Scientific Notation	Name
Exa (E)	18	1,000,000,000,000,000,000	10^{18}	quintillion
Peta (P)	15	1,000,000,000,000,000	10^{15}	quadrillion
Tera (T)	12	1,000,000,000,000	10^{12}	trillion
Giga (G)	9	1,000,000,000	10^{9}	billion
Mega (M)	6	1,000,000	10^{6}	million
kilo (k)	3	1,000	10^{3}	thousand
hecto (h)	2	100	10^{2}	hundred
deca (da)	1	10	10^{1}	ten
	0	1	10^{0}	one
deci (d)	-1	0.1	10^{-1}	one tenth
centi (c)	-2	0.01	10^{-2}	one hundredth
milli (m)	-3	0.001	10^{-3}	one thousandth
micro (μ)	-6	0.000001	10^{-6}	one millionth
nano (n)	-9	0.000000001	10^{-9}	one billionth
pico (p)	-12	0.000000000001	10^{-12}	one trillionth
femto (f)	-15	0.000000000000001	10^{-15}	one quadrillionth
atto (a)	-18	0.000000000000000001	10^{-18}	one quintillionth