Math 5e, Fall 2024 Homework 9

Homework #8 is due November 20

Instructions: Some of the problems we solved in class, and some are new. Please try to solve all problems, do your best, and show your work. Write on separate sheets of paper, not between the lines of this handout!

General notation (*n* is a whole number):

$$a^n = a \times a \times a \times \ldots \times a \text{ (}n\text{ times)}$$

Special cases:

$a^0 = 1$	read: *a*-to-the-zero
$a^1 = a$	is just itself '*a*'
$a^2 = a \times a$	read: *a*-squared
$a^3 = a \times a \times a$	read: *a*-cubed

1. Solve the following equations:

   (a) $5(x-1) - 4 = 3x + 1$
   (b) $\frac{2}{3}(x-2) = -18$
   (c) $|2x + 1| = 7$

2. When Dennis was 27, his son was three years old. Now, his son's age is one-third of Dennis' age. How old is each of them now?

3. Find the sum of $1 + 2 + 4 + \cdots + 2^n$ (the answer will depend on *n*). [Hint: first try computing it for several small values of n, see if a pattern emerges, then find a general rule.]

4. If we put one grain of wheat on the first square of the chessboard, two on the second, then four, eight, … approximately how many wheat grains will there be? (You can use $2^{10} = 1024 \approx 1000$.)

5. Lotus flowers are growing in a lake. Every day, each lotus plant divides into two plants, so the area its leaves cover is doubled. In 30 days the whole lake is covered with lotus leaves. When did leaves cover exactly half of the lake?

6. There are 15 samples of water from various wells. It is known that exactly one of them contains a dangerous chemical. A lab can test for the chemical, but the analysis is time-consuming and expensive. Can you find the sample containing the chemical using fewer than 15 tests? How many tests are needed? [Hint: take a droplet of water from each sample and test the new combined sample … use the strategy in 'example 2' above.]

7. Write as powers with base 2:
   (a) 32×2^3
   (b) $2^5 \times 2^3 \times 16$
   (c) $2^3(16 + 2^4)$

8. Find the prime factorization of 500 and 1215. Express as a multiplication of powers and find the greatest common factor (GCF).