MATH 5e: Class Work 3

Topics

- Algebraic expressions and Variables

In mathematics and other sciences, we often use letters instead of numbers. These letters are called ***variables.*** Letters can mean that there is a group of numbers that can be used instead of the letter. Letters are also commonly used to denote an unknown value.

Expressions involving both numbers and variables are called ***algebraic expressions***. In algebraic expressions we omit the sign of multiplication between a number and a variable. Instead of $7\times b$ we write $7b$, instead of $10\times z$ we write $10z$. In the products, a number goes first, and then the variable(s). The number is called a coefficient. We do not write $k\times 10$, we write $10k$, where 10 is the coefficient and k is the variable.

- Arithmetic rules when simplifying expressions

Using variables, we can write the basic rules for addition and multiplication as follows:

$a + b = b + a$	commutative law for addition
$a + (b + c) = (a + b) + c$	associative law for addition
$ab = ba$	commutative law for multiplication
$a(bc) = (ab)c$	associative law for multiplication
$a(b + c) = ab + ac$	distributive law

These laws can be used to simplify calculations and rewrite expressions in a simpler form.
For example:

$2x + 3 + 5 \times (x + 1)$	$= 2x + 3 + 5x + 5$	"opening parentheses"
	$= 2x + 5x + 3 + 5$	"finding like terms"
	$= (2 + 5)x + 8$	"combining terms"
	$= 7x + 8$	

We continue the solution on subsequent lines with '=' sign. The operation we did in the last line – combining terms $2x$ and $5x$ into a single term $7x$ – is very commonly used; it is called "collecting-like terms." It is only possible if the terms contain the same variable: we cannot collect like terms in an expression with different variables, for example, $2x + 7y$.

- Algebraic expressions with negative numbers

Opening parenthesis, addition, and subtraction

$+(-a) = -a$ $\quad$ $(-a) + (-b) = -a - b = -(a + b)$

$-(-a) = +a$ $\quad$ $(-a) + (+b) = -a + b = +(b - a)$ $\quad if\ |b| > |a|$

$= -(a - b),\ if\ |a| > |b|$

We can "open the brackets" in the following expressions:

$$a - (b + c) = a - b - c$$
$$a - (b - c) = a - b + c$$
$$a(b - c) = ab - ac$$

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- Solving simple equations

An *equation* has expressions on both sides of the equal sign. The letters *x, y,* and *z* represent the unknown variables we are trying to find. Given an equation, we can add or subtract the same number from both sides. For example:

$3x + 5 = 20$ subtracting 5 from both sides of the original equation

$3x + 5 - 5 = 20 - 5$

$3x = 15$

We can multiply or divide both sides of an equation by the same number.

$3x = 15$ dividing both sides by 3

$3x \div 3 = 15 \div 3$

$x = 5$

- Expressions with fractions

The division of two numbers can be represented as a fraction.

$a \div b = \frac{a}{b}$ substitute ÷ for the fraction bar.

A *compound fraction.* In an expression with many "÷" signs, where all ÷ signs are substituted with the fraction bar, for example:

$$\big((y+1) \div b + 1\big) \div (x+3) \div d = \left(\frac{\frac{y+1}{b} + 1}{(x+3) \div d}\right)$$

Problems

1. Which is the variable, which is the coefficient in the following expressions

$3a;$ $7b + 8;$ $-5\frac{1}{3}b;$ $a \times 3;$ $-2ab \times \frac{1}{2};$ $-a$

2. Find the values of the algebraic expressions:

(a) $78 + 3x$ for $x = 8$ and 23;

(b) $54 \div (x - 7)$ for $x = 13$ and 11;

3. Simplify the expressions

a) $2x + 7 + 5x + +2 + 3x =$

b) $3x + 9 + 5xy + 2xy + 3 =$

c) $2a + 1 + 3(a + 2) =$

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4. Review addition and subtraction with negative numbers. Simplify the expressions

   a) $-7-(-9)=$

   b) $-\big(-6+(-4)\big)=$

   c) $-3-\big(7+(-6)\big)=$

   d) $-3-(-4)+(-5)=$

   e) $-(-(+2)+5)=$

5. Rewriting expressions in a simpler form.

   a) $2x+3+5\times(x+1)=$

   b) $3a+2(a+2b)-2b=$

   c) $3(x-3y)-y=$

   d) $a-(a+2b)-2b=$

6. Solve the following equations.

   a) $x+12=34$
   b) $24-x=10$
   c) $5(x-2)=25$
   d) $4x=2x+8$
   e) $(-2)\times x=-7$
   f) $(-3)\times x+2=x-18$

7. Write the following expressions as composite fractions:

   a) $a\div b\div c=$

   b) $d\div c+b\div c=$

   c) $(x-a\div 4)+x\div 4=$

   d) $a\times 4-(5+x)\div c=$

   e) $(a+c-1)\div(a-c)=$