MATH 5e: Class Work 7

Topics: Fractions and decimals. More word problems

- Sets

Sets are collections of similar elements. In mathematics, we usually talk about sets of numbers. The main sets are:

• N: Set of natural numbers, i.e., $1, 2, 3, \ldots$;
Operations: $+, \times$.

• Z: Set of integer numbers, i.e. $\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots$;
Operations: $+, -, \times$.

• Q: Set of rational numbers, i.e., numbers that can be written as a fraction
Operations: $+, -, \times, \div$.

- Fractions to decimals – using long division

We keep doing long division to convert fractions to decimals until the remainder repeats. Every fraction can be converted to a decimal, which will either be finite or it will be infinite, but there will be a sequence of numbers which constantly repeats itself.

$\frac{2}{7} = 0.285714285714 \cdots = 0.285714.$

```
   0.2857142
7 | 2.0000000
    1.4
    ---
     60
     56
     --
     40
     35
     --
     50
     49
     --
     10
      7
      --
      30
      28
      --
      20
```

- Quantities denoted with fractions

Rate: In math and science, a rate is a quantity or amount measured in relation to another quantity or amount. It is usually presented as a fraction. Rates are used in many everyday situations, such as calculating how fast someone is driving or the interest rate on a savings account.

Example: 60 mi/1h - speed is a rate of distance and time
3 hours to deliver 180 newspapers – rate is 180newspapares/3h

Unit rate: is a rate with a denominator of one.

Example: 60 notebooks cost \$30 - the unit rate is \$0.50/per notebook

Ratio: a ratio is the quotient of two quantities of the same type, such as volume and volume.

Example: the bottle is ¾ full - ratio of the volume of the bottle to the volume of the water

Proportion: the division of two numbers $\frac{a}{b}$ where $b \neq 0$

Proportion is also an equality of the type $\frac{a}{b} = \frac{c}{d}$ that can also be written as $a \cdot d = b \cdot c$

The number k = $\frac{a}{b}$ is known as the coefficient of proportionality

Example: In a map, 1 cm corresponds to a distance of 25 km. These numbers form the ratio 1cm: 25 km. How many cm on the map is a distance of 75 km?

$\frac{1cm}{15km} = \frac{xcm}{75km}$ then $1cm \cdot 75\ km = x\ cm\ \cdot 15\ km$, so $x\ cm = \frac{1cm \cdot 75\ km}{15\ km} = 5\ cm$

MATH 5e: Class Work 7

Problems

1. A messenger was sent from one city to another that is many kilometers away. He can travel 40 km in one day. Another messenger was set a day after. He can travel 45 kilometers in one day. After how many days will the second messenger overtake the first one? Hint: create an equation where the unknown is the number of days.

2. A pen is three times as expensive as a notebook. If Bill buys 5 pens, he will have 6 dollars left. If Bill buys 3 pens and 3 notebooks, he will have 12 dollars left. How much extra money will he need if he decides to buy 7 pens and 4 notebooks?

3. Two secretaries, Barbara and Mary, need to type a 100-page document. Barbara can type it in 4 hours, but Mary types slower, so it would take her 5 hours to do this. If they divide the work between the two of them in the most efficient way, how fast can they type it together?
   Hint: how many pages does each secretary type in 1 h?

4. Convert the following fractions into decimals:
   a) $\frac{1}{8}$ b) $\frac{5}{6}$ c) $\frac{1}{13}$

5. Check if the following ratios form the same proportions. If yes, find the proportionality coefficient
   a) $\frac{4}{6}$ and $\frac{10}{15}$ b) $\frac{7}{21}$ and $\frac{3}{12}$ c) 11 and $\frac{33}{7}$

6. A car traveled from Chicago to Detroit, starting with a full tank of gas. When the driver arrived in Detroit, she found she had traveled 450 km. When she filled the tank again to the top, she added 30 liters of gas. Calculate to the nearest hundredths how many liters per 100 km her car used on this trip.

7. On a production line with 16 machines, one client's order could be fulfilled in 30 hours. 4 of the machines broke down. How long will it take to fulfill the order with the rest of the machines?

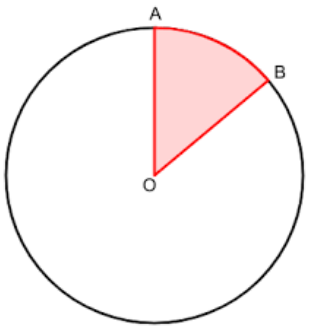
8. A mountain climber discovered that above 2000 m, where the temperature is 0ºC, the temperature decreased proportionally, with 1ºC every 200 m.
   a) Determine the proportionality coefficient.
   b) Graph the temperature t (in ºC) versus height (in m) using the points for $x = 200, 600,$ and 800 m above 2000 m. Take 1 division on the graph to be 100 m.

A sector is a part of a circle enclosed between 2 radii. A central angle is the ∢ AOB with a vertex at the circle's center. The size of a sector is proportional to the measure of the angle.
Example: a central angle that is 90º is 90º/360º = ¼ of the circle.

Central angles can be used to present data in a circular diagram – a pie chart.

MATH 5e: Class Work 7

9. Find what part of the circle is a central angle with a measure of
   a) 60^o
   b) 45^o
   c) 150^o



10. In one school, 48 students play varsity sports. Four students are wrestling, eight play basketball, sixteen play soccer, twelve compete in track and field, and eight play volleyball.
    Represent the students by sport using a pie chart.