CLASS WORK 12,
January 17, 2021

Power properties:

$$(ab)^n = \underbrace{ab \cdot ab \cdots ab}_{n\ times} = \underbrace{a \cdot a \cdots a}_{n\ times} \cdot \underbrace{b \cdot b \cdots b}_{n\ times} = a^n b^n$$

$$a^m a^n = \underbrace{a \cdot a \cdots a}_{m\ times} \cdot \underbrace{a \cdot a \cdots a}_{n\ times} \cdot = \underbrace{a \cdot a \cdots a}_{m+n\ times} = a^{m+n}$$

$$\frac{a^m}{a^n} = \frac{\overbrace{a \cdot a \cdots a}^{m\ times}}{\underbrace{a \cdot a \cdots a}_{n\ times}} = \underbrace{a \cdot a \cdots a}_{m-n\ times} = a^{m-n} \qquad if\ m > n$$

$$\frac{a^m}{a^n} = \frac{\overbrace{a \cdot a \cdots a}^{m\ times}}{\underbrace{a \cdot a \cdots a}_{n\ times}} = \frac{1}{\underbrace{a \cdot a \cdots a}_{n-m\ times}} = \frac{1}{a^{n-m}} \qquad if\ m < n$$

$$a^{-1} = \frac{1}{a}$$

$$a^{-3} = \frac{1}{a \cdot a \cdot a} = \frac{1}{a^3}$$

$$a^{-n} = \frac{1}{a^n}$$

Write the following expressions as powers:

a. $\frac{1}{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2} =$

b. 1mm = ? m

c. 1dm = ?m

d. 1km = ? m

e. $\frac{a}{a \cdot a \cdot a \cdot a \cdot a \cdot a \cdot a \cdot a \cdot a} =$

MATH 5, HOMEWORK 12,

January 12, 2025

1. Write the following expressions as powers:
   f. $\frac{1}{2\cdot2\cdot2\cdot2\cdot2\cdot2} =$
   g. 1mm = ? m
   h. 1dm = ?m
   i. 1km = ? m
   j. $\frac{a}{a\cdot a\cdot a\cdot a\cdot a\cdot a\cdot a\cdot a\cdot a} =$

2. Simplify:
   a. $2^7 \cdot 5^3 \cdot 2^3 \cdot 5^7 =$
   b. $\frac{2^7\cdot 2^7}{2^3\cdot 2^4} =$
3. Solve the following equations:
   a) $2(x - 1) = \frac{2}{3}(x + 5)$
   b) $2x - 5(x - 7) = -1$
   c) $\frac{x-2}{x-1} = 3$

4. Do the following arithmetic operations with binary numbers. Do them without converting the numbers to decimal form.
   (a) $110101_b + 111011_b$
   (b) $10101_b \times 1011_b$
   (c) $(10101_b + 1101_b) \times 10110_b$

5. The following is a beginning of a computer file. Can you decode it (assuming it is written in the standard, Latin 1, encoding)?

01010100 01101111 01110000 00100000 01110011 01100101 01100011 01110010 01100101 01110100 00001010

6. Fish head weighs as much as the tail and half of the body together. The body weighs as much head and tail together, and the tail weighs 1 kg. How heavy is the fish?
7. You are given several coins, one of which is fake. The weight of the fake one is different from the weight of the real ones, but it is not known whether it is heavier or lighter. Can you find whether the fake one is heavier or lighter than the real one using two measurements with the scales (2 platforms, no weights) if the total number of coins is:
   a) 100
   b) 99
   c) 98
   You do not have to find which coin is the fake one.
8. "Jack has at least a thousand books", said Maria. "No, he has less than a thousand", said Daniel. "He certainly has at least one book", said Kathy. If it is known that only one of the statements is true, how many books does Jack have?