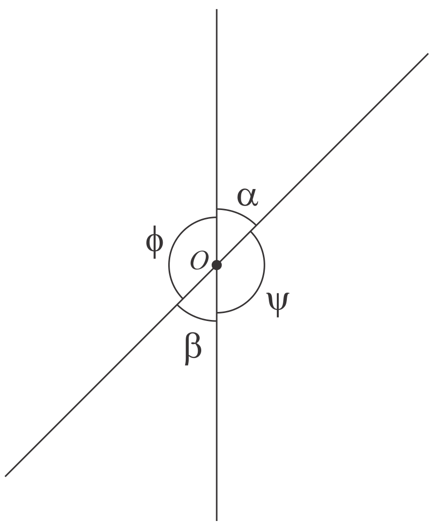
Classwork 13.

Geometry.

When two straight lines intersect at a point, four angles are formed. A pair of angles opposite each other formed by two intersecting straight lines that form an "X"-like shape, are called vertical angles, or opposite angles, or vertically opposite angles.



α and β and ϕ and ψ are 2 pairs of vertical angles.

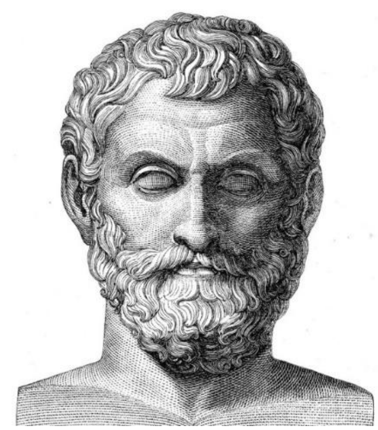
***Vertical angles theorem**:*

Vertical angles are equal.

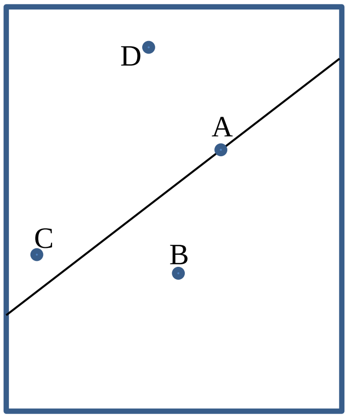
In mathematics, a **theorem** is a statement that has been proven on the basis of previously established statements.
According to a historical legend, when Thales visited Egypt, he observed that whenever the Egyptians drew two intersecting lines, they would measure the vertical angles to make sure that they were equal. Thales concluded that one could prove that vertical angles are always equal and there is no need to measure them every time.

Proof:
$\angle\phi + \angle\alpha = 180°$ because they are supplementary by construction.
$\angle\phi + \angle\beta = 180°$ because they are supplementary also by construction.
$\Rightarrow \angle\alpha = \angle\beta$, therefore we proved that if 2 angles are vertical angles then they are equal. Can we tell that invers is also the truth? Can we tell that if 2 angles are equal than they are vertical angels?

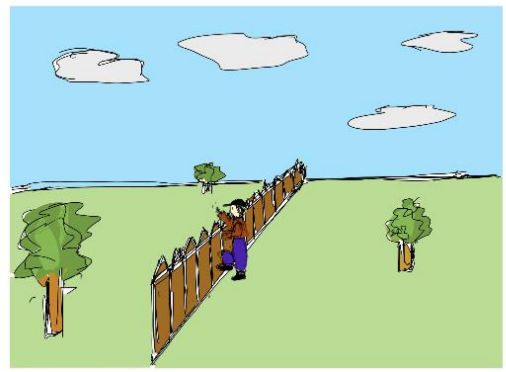
(**Thales of Miletus** 624-546 BC was a Greek philosopher and mathematician from Miletus. Thales attempted to explain natural phenomena without reference to mythology. Thales used geometry to calculate the heights of pyramids and the distance of ships from the shore. He is the first known individual to use deductive reasoning applied to geometry, he also has been credited with the discovery of five theorems. He is the first known individual to whom a mathematical discovery has been attributed (Thales theorem).



Take a sheet of paper and draw a straight line. Use ruler. Draw the line from one side of paper to the other. This line divides a plane into two parts, two half-planes. Both, plane and line have no ends.

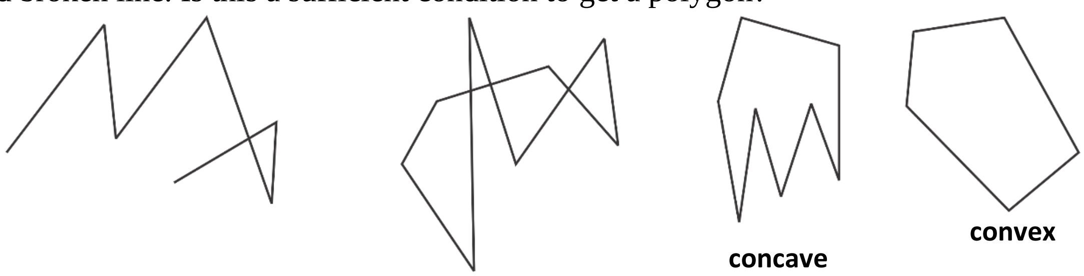


If a point is marked on this plane, the point can belong to the line or not. Points C and D belong to the same half-plane, they can be connected without crossing the line. Points C and B belong to two different half-planes, they can't be connected without crossing the line.



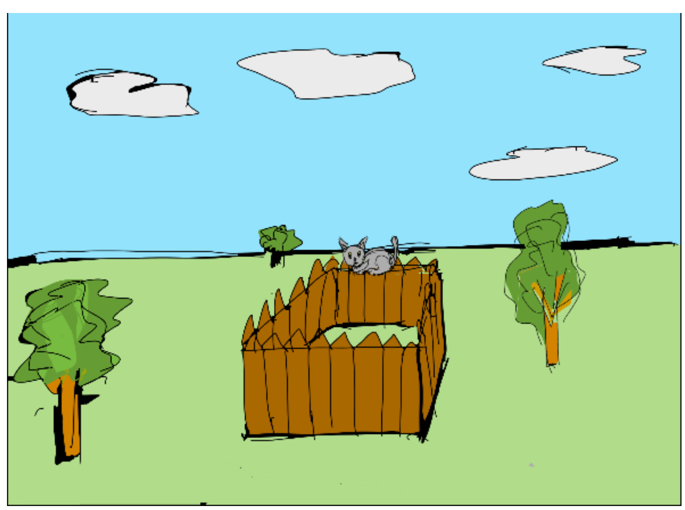
Polygons.

Draw a chain of segments, so that the last point of one segment is a first point of the next, and three consecutive points don't lie on the same line.
Draw such chain so that the last point of the last segment is the first point of the first one. We got a closed broken line. Is this a sufficient condition to get a polygon?



- Polygon is a closed figure (area), two points inside of the polygon can be connected without crossing the boundaries, but one point inside and one outside can't be connected without crossing the edge of the figure.

  The simplest polygon is a triangle.
  Draw a triangle. Measure its angles. Add them together. What number did you get?



Exercises:

1. On a line mark two points. How many segments were formed? Add one point. How many segments are there now? Add one more point. How many segments are there now? How many segments 6 points will form on the line? 10? 99?

2. Into how many parts two lines can divide a plane?

3. 3 lines can divide a plane? 4 line? 5 lines?

4. Into how many parts three rays can divide a plane?

5. Into how many parts two angles can divide a plane?

6. Mark 2 points. How many different lines can be drawn through these two points?

7. Mark three points. How many lines can be drawn through three points?
   Consider all possible solution.

8. Mark four points. (Any three points do not belongs to the same line). How many lines can be drawn through four points?10 points? 100 points? N points?

9. How many intersections produce two non-parallel lines? Three non-parallel lines? 10 lines? 100? N lines?

10. 4 angles are formed at the intersection of 2 lines. One of them is 30°. What is the measure of 3 others?

11. Do the operations with angular measures:
    $a.\ 25^\circ\,36'\,24'' + 36^\circ 24'\,40''$ $b.\ 48^\circ 26' + 28^\circ 36'\,34''$
    $c.\ 48^\circ 48' 48'' - 24^\circ 36' 36''$ $d.\ 3 \cdot 24^\circ 36'$
12.