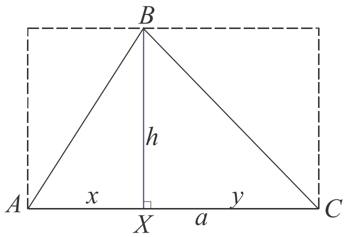
Math 5b. Classwork19

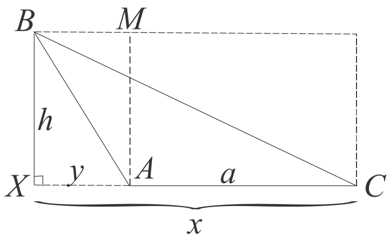


The area of a triangle is equal to half of the product of its altitude and the base, corresponding to this altitude.

For the acute triangle it is easy to see.

$S_{rec} = h \cdot a = h \cdot (x + y) = hx + hy$

$S_{\Delta ABX} = \frac{1}{2} h \cdot x, \qquad S_{\Delta XBC} = \frac{1}{2} h \cdot y, \qquad S_{\Delta ABC} = S_{\Delta ABX} + S_{\Delta XBC}$

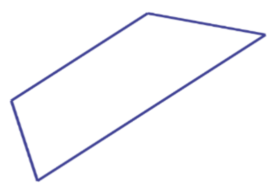


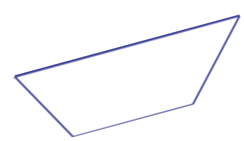
$$S_{\Delta ABC} = \frac{1}{2} h \cdot x + \frac{1}{2} h \cdot y = \frac{1}{2} h (x + y) = \frac{1}{2} h \times a$$

For an obtuse triangle it is not so obvious for the altitude drawn from the acute angle vertex. Can you come up with the idea how we can prove it for an obtuse triangle? Area of trapezoid?

Quadrilaterals.

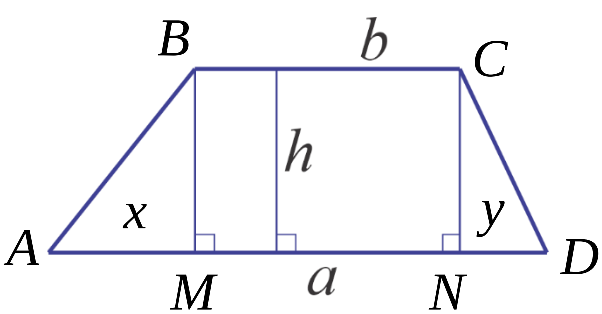
Polygons with four sides and four vertices are quadrilaterals. Quadrilaterals can have four non parallel sides, two parallel and two not parallel sides, and two pairs of parallel sides.





If the quadrilateral has only one pair of parallel sides and two other sides are not parallel are called trapezoids.

Trapezoid has two bases, a and b, they are parallel segments. h is an altitude (height), segment, perpendicular to bases. How to find area of the trapezoid? MBCN is a rectangle, area of this rectangle is

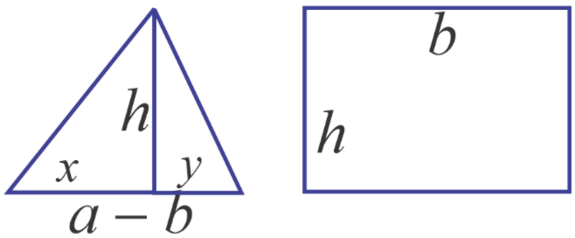


$$S_{rectangle} = h \cdot |MN| = h \cdot b$$

Area of the trapezoid is

$S = S_{rectangle} + S_{AMB} + S_{NCD}$

$S_{AMB} + S_{NCD} = \frac{1}{2} \cdot h \cdot (x + y) = \frac{1}{2} \cdot h \cdot (a - b)$



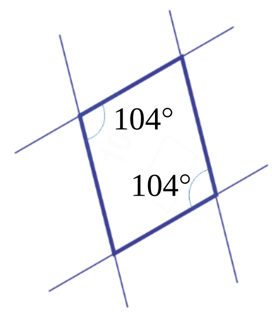
$$S = S_{rectangle} + S_{AMB} + S_{NCD}$$

$= hb + \frac{1}{2}h(a-b) = hb + \frac{1}{2}ha - \frac{1}{2}hb = \frac{1}{2}hb + \frac{1}{2}ha = \frac{1}{2}h(a = b);$

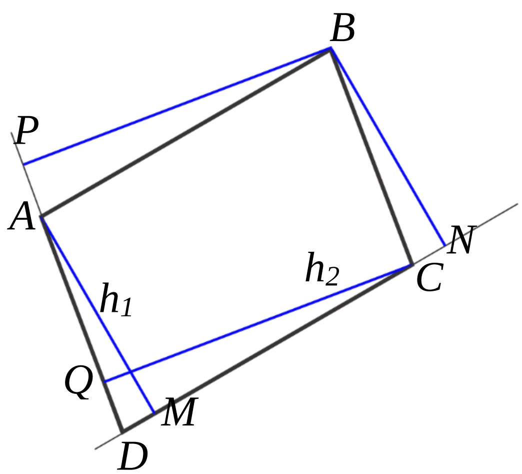
Area of a trapezoid is a half of the product of the altitude and the sum of the bases.

If a rectangle has two pairs of parallel lines it's called a parallelogramm. Parallelogramms havs e few properties:

- Their sides not only parallel, but also equal.
- Diagonal divides a parallelogram onto two equal (congruent) triangles.
- Diagonals intersect at the midpoint.
- Opposite angles are equal.



How do we call a parallelogram with all right angles? Parallelograms with equal sides are called rhombuses.



Area of a parallelogram. On the picture below. ABCD is a parallelogram. Segments [AM] and [BN] are equal and perpendicular to lines (DC) and (AB). Triangles DAM and CBN are equal. You can see it by superimposing them (and it can be proved based on the theorems of triangle equalities). So the area of parallelogram is equale to the area of a rectangle

$$S_{ABNM} = |MN| \cdot h_1 = |DC| \cdot h_1$$

(h_1 is an altitude, distance between a pair of parallel lines. Of course, it's also equal to

$$S_{ABNM} = |AD| \cdot h_2$$