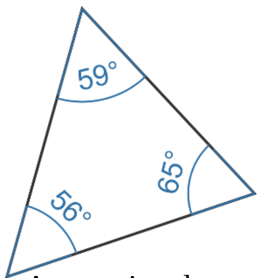
Math 5b. Classwork17

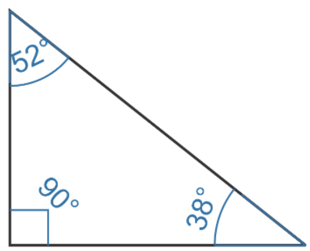
The simplest polygon is a triangle. How we can classify triangles?

- By angles: triangles can be acute, obtuse, and wright triangles.

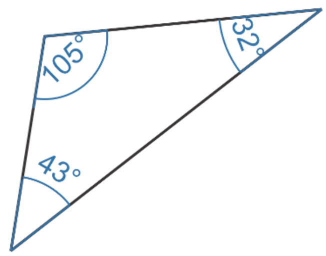
  Acute triangle has all three acute angles. (Acute angle is an angle less then wright angle).



Acute triangle.



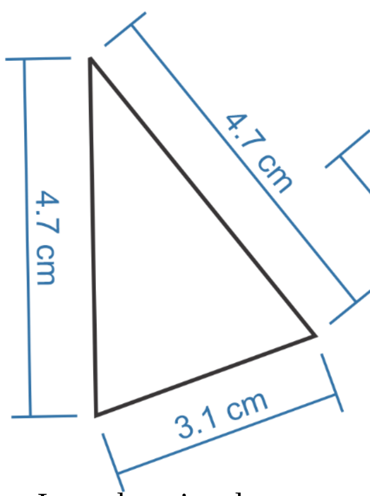
Wright triangle.



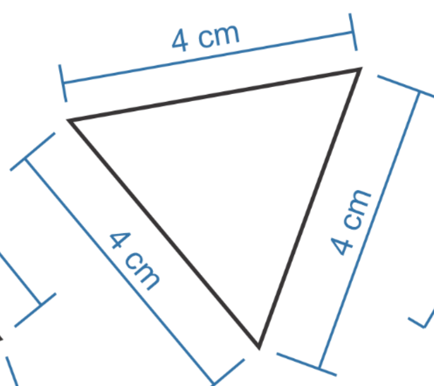
Obtuse triangle.

  Obtuse triangle has one obtuse angle. Wright triangle has a wright angle.

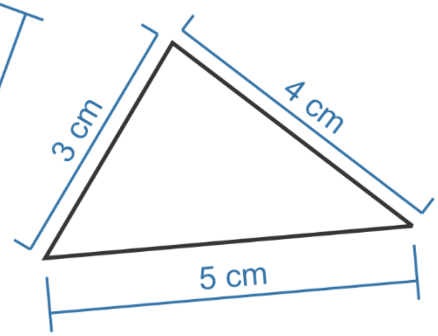
- By sides: isosceles triangle, equilateral triangle, and scalene triangle. Isosceles triangle has two equal sides, equilateral triangle has all tree equal sides, and scalene triangle, with all three different sides.



Isosceles triangle.

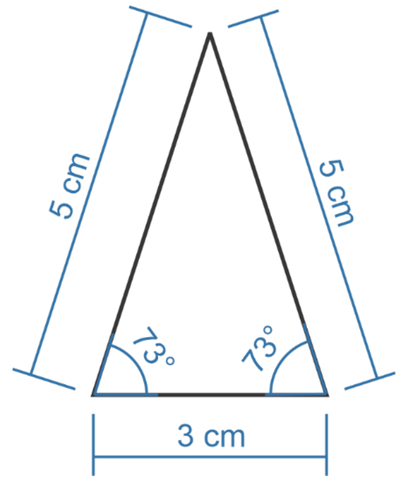


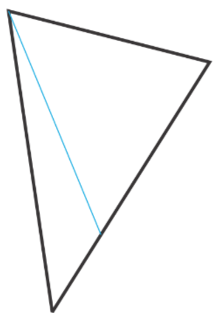
Equilateral triangle.



Scalene triangle.

We know that bigger angle is opposite to the longer side. (This is a theorem, but we are not going to prove it for now). Base on this fact, it would be logical to suppose that in equilateral triangle all three angles are equal, and in isosceles triangle angels adjoined to the base are equal.

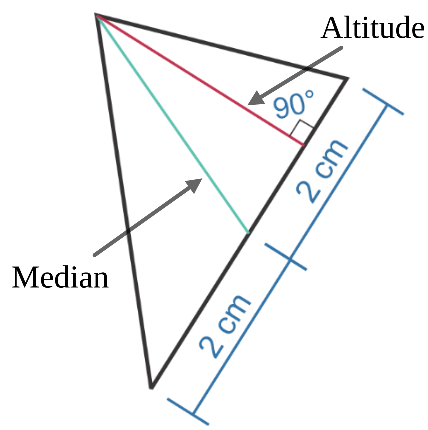


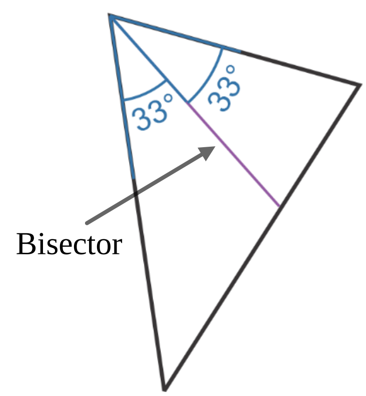


In each triangle we can connect a vertex with a point on the opposite side, like on the picture. Infinitely many segments can be drawn from any vertex to an opposite side, but there are three very special segments. From each vertex one can draw a segment to a midpoint of the opposite side, such segment is called a ***median***.

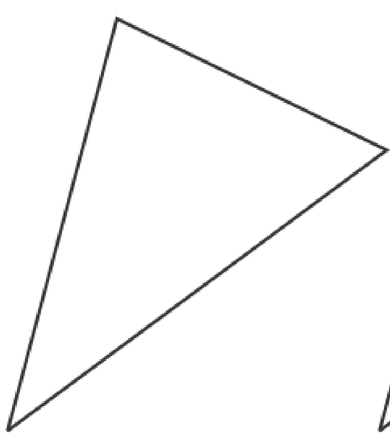
Also, from any vertex one can draw a perpendicular to an opposite side (or to continuation of the opposite side in the case of obtuse angle). This segment is called an ***altitude*** (or height).

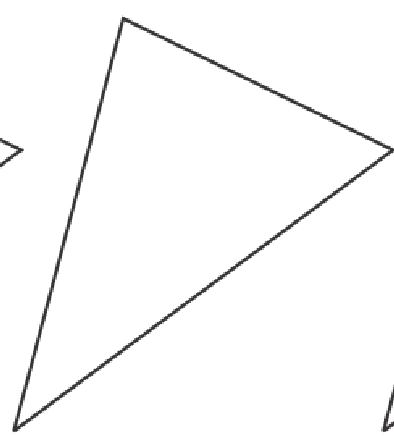
And the last segment is an (angular) ***bisector***. It’s a segment, drawn from any vertex of a triangle, in a way that the angel is divided into two equal angles.

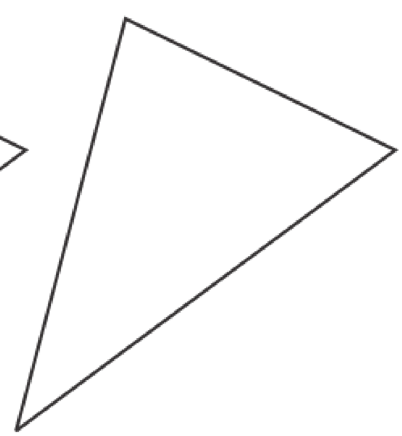


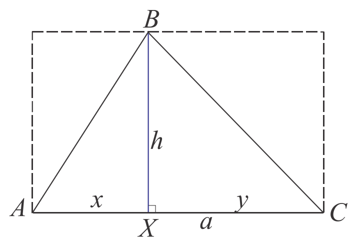


Draw medians, altitudes and bisectors in the triangles:







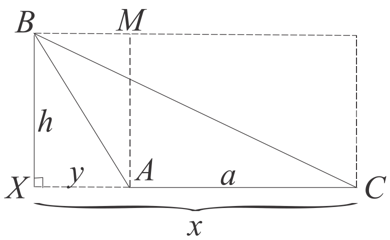


The area of a triangle is equal to half of the product of its altitude and the base, corresponding to this altitude.

For the acute triangle it is easy to see.

$S_{rec} = h \cdot a = h \cdot (x + y) = hx + hy$

$$S_{\Delta ABX} = \frac{1}{2} h \cdot x, \qquad S_{\Delta XBC} = \frac{1}{2} h \cdot y, \qquad S_{\Delta ABC} = S_{\Delta ABX} + S_{\Delta XBC}$$

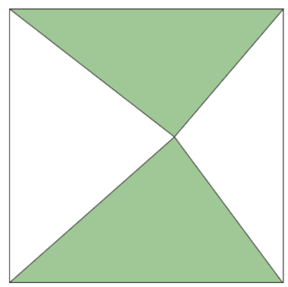


$$S_{\Delta ABC} = \frac{1}{2} h \cdot x + \frac{1}{2} h \cdot y = \frac{1}{2} h(x + y) = \frac{1}{2} h \times a$$

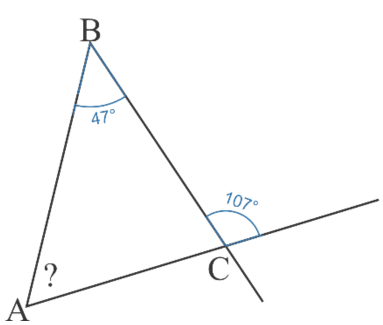
For an obtuse triangle it is not so obvious for the altitude drawn from the acute angle vertex. Can you come up with the idea how we can prove it for an obtuse triangle? Area of trapezoid?

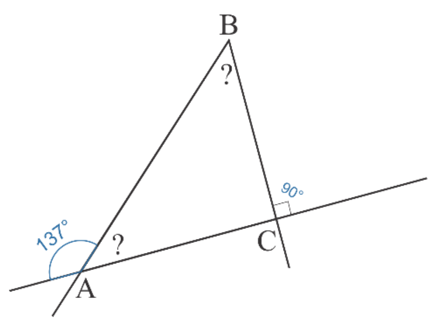
Exercises:

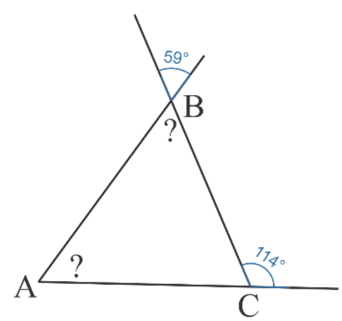
1. Which part of the square is shaded?

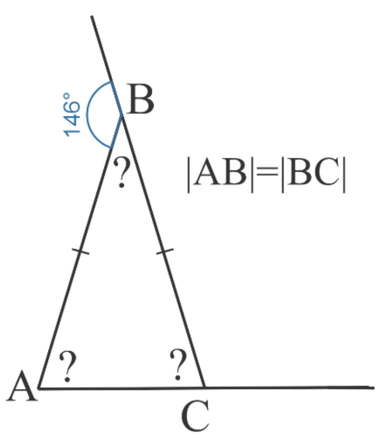


2. Find angles:









3. Find the sum of

$$\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\frac{1}{4\cdot 5}+\cdots+\frac{1}{2024\cdot 2025}$$

4. Write without parenthesis;

   $a.\ (x-1)(x+1);$ $b.\ (x-1)^2;$

5. Using the result from the previous problem, reduce the fraction:

$$\frac{x^2-2x+1}{x^2-1}; \qquad x \neq 1, -1$$