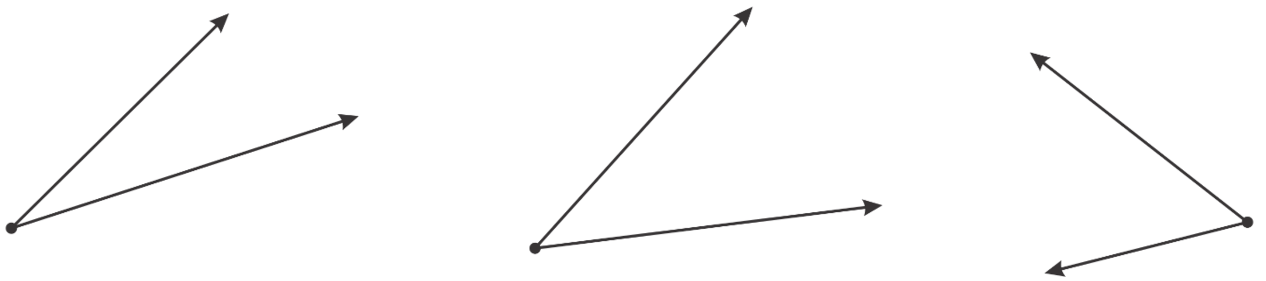
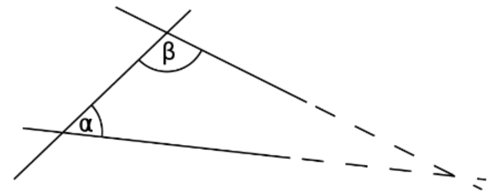
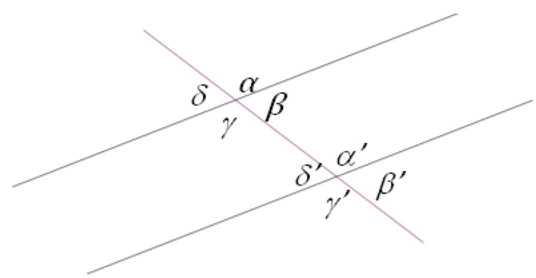
Math 5b. Classwork16



Two of the most important building blocks of geometric proofs are axioms and postulates. Axioms and postulates are essentially the same thing: mathematical truths that are accepted without proof. Their role is very similar to that of undefined terms: they lay a foundation for the study of more complicated geometry. One of the greatest Greeks (*Euclid*) achievements was setting up such rules for plane geometry. This system consisted of a collection of undefined terms like point and line, and five axioms from which all other properties could be deduced by a formal process of logic. Four of the axioms were so self-evident that it would be unthinkable to call any system a geometry unless it satisfied them:



1. A straight line may be drawn between any two points.
2. Any terminated straight line may be extended indefinitely.
3. A circle may be drawn with any given point as center and any given radius.
4. All right angles are equal.

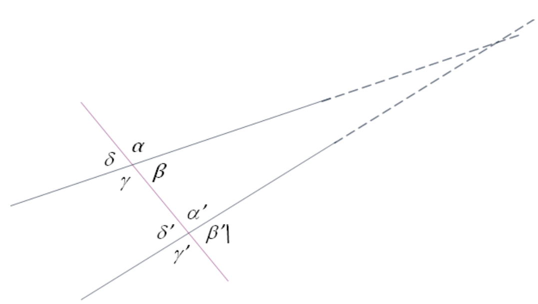


But the fifth axiom was a different sort of statement:

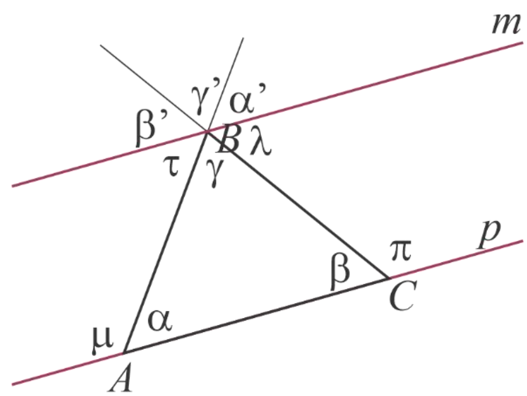
5. If two straight lines in a plane are met by another line, and if the sum of the internal angles on one side is less than two right angles, then the straight lines will meet if extended sufficiently on the side on which the sum of the angles is less than two right angles.

Angles formed when transversal crosses two parallel lines:

On this picture $\beta + \alpha' < 180°$, so lines are not parallel and will intersect at the side of angles β and α', if extended. $\gamma + \delta' > 180°$ (greater than strait angle), lines will not intersect at that side.



The fifth axiom along with the theorem about the vertical angles is a base for the proof of the several statements about interior and exterior angles. For example, let’s prove the theorem about the sum of the angles of a triangle.



Theorem: the sum of the angles of a triangle is a straight angle.

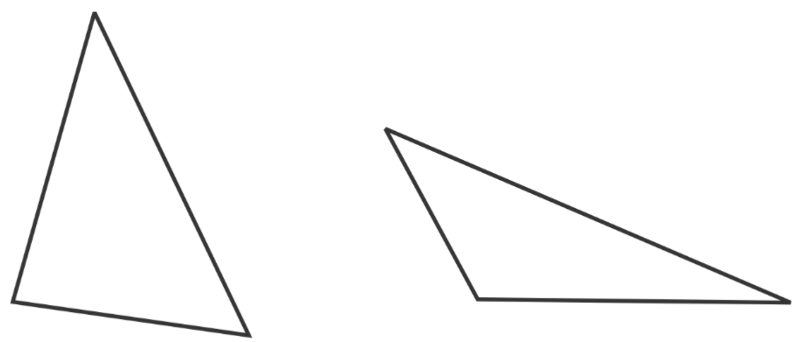
Triangle ABC has angles α, β, and γ. Segment [AC] belongs to line p ($[AB] \in p$).

Let’s draw the line m, parallel to line p and passing through he point B ($p||m$). Angles μ and τ and π and λ are supplementary, because the lines m and p parallel, 5th axiom.

Angles μ and α are also supplementary by construction. Now we can tell that angle τ and angle α are equal. Angle τ and angle α’ are vertical, therefore they are equal. Finally, we can say that angle α and α’ are equal. Same conclusion can be derived for angles β and β’. Angles γ and γ’ are vertical, therefore are equal. Altogether, angles α’, β’, and γ’ form a straight angle. Angles α, β, and γ equal to α’, β’, and γ’, so they also sum up to a straight angle.

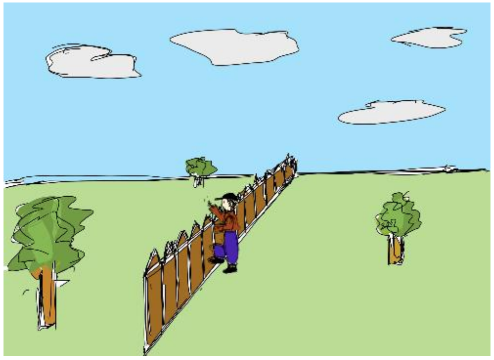
Let’s check it!

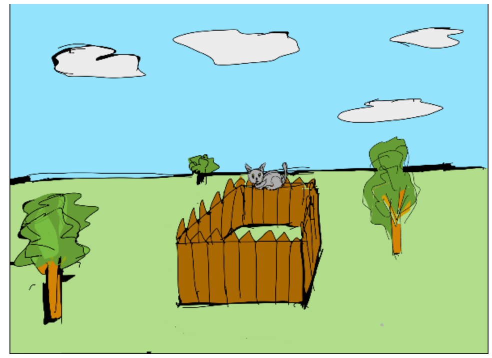
Measure the angles of triangles, add the measurements. What did you get?



Polygons.

Draw a chain of segments, so that the last point of one segment is a first point of the next, and three consecutive points don’t lie on the same line.
Draw such chain so that the last point of the last segment is the first point of the first one. We got a closed broken line. Is this a sufficient condition to get a polygon?

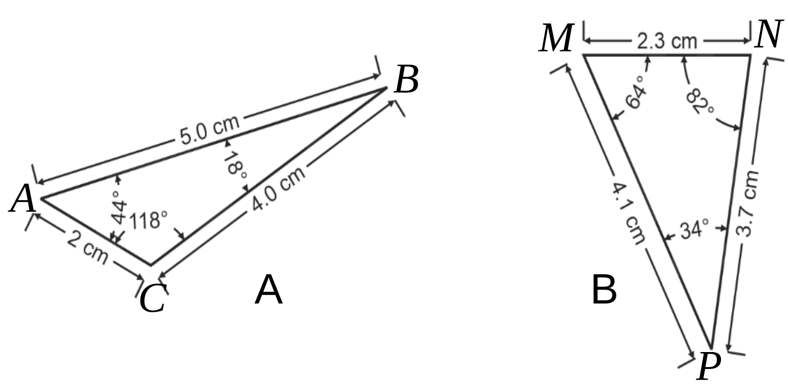




A **polygon** is a closed figure (an area) where two points inside the polygon can be connected without crossing the boundaries, but a point inside and a point outside cannot be connected without crossing the edge of the figure.

The simplest polygon is a **triangle**. We already know that the sum of the angles of a triangle equals a straight angle (180°).

What else can we say about a triangle? Each triangle has three sides, or segments. Can any three arbitrary segments form a triangle?



For any triangle, **any side must be smaller than the sum of the other two sides**.

The bigger side is opposite to the bigger angle.

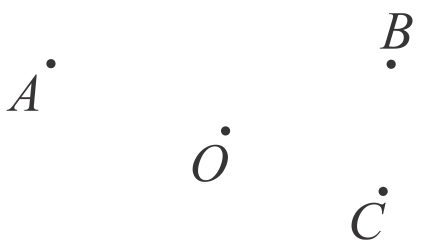
Triangle	Angle	Opposite side
ABC	$\angle ABC = 18^{\circ}$	$\lvert AC \rvert = 2\ cm$
	$\angle CAB = 44^{\circ}$	$\lvert AC \rvert = 4\ cm$
	$\angle ACB = 118^{\circ}$	$\lvert AC \rvert = 5\ cm$
MNP	$\angle MPN = 34^{\circ}$	$\lvert AC \rvert = 2.3\ cm$
	$\angle PMN = 64^{\circ}$	$\lvert AC \rvert = 3.7\ cm$
	$\angle MNP = 82^{\circ}$	$\lvert AC \rvert = 4.1\ cm$

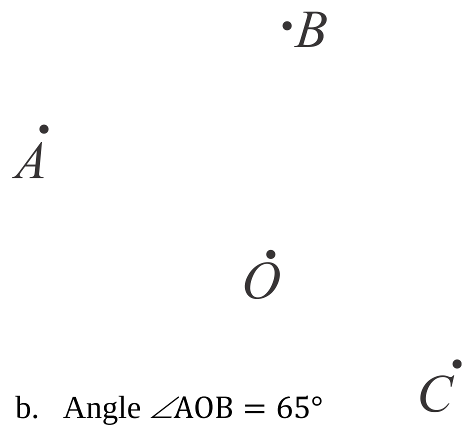
Exercises:

1. Draw a triangle with sides 3 cm, 5 cm and the angle between them 50°.

2. Draw a triangle with angles 30° and 50° and the side between them 7 cm. Do we need another information?

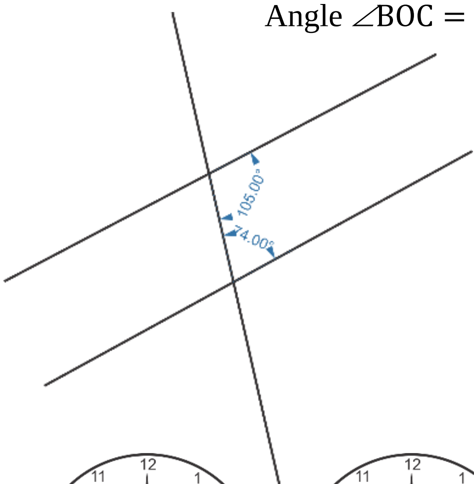
3. Do the points A, O, and C lie on the same line?





a. Angle $\angle AOB = 137°$
   Angle $\angle BOC = 43°$

b. Angle $\angle AOB = 65°$
   Angle $\angle BOC = 116°$



4. Are these two lines parallel?

5. What is the angle between the small and big hands of a watch at the time: