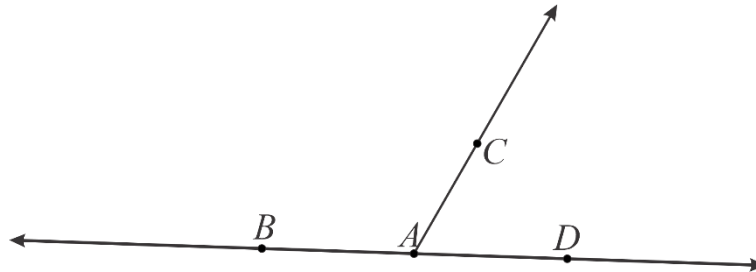


## Math 5b, classwork 15.

If a point marked on a line, it produces two rays with the common vertex, an angle. This angle has it's own name: a straight angle.



If another ray is coming from the vertex of a straight angle, we now have three angles,  $\angle CAB$ ,  $\angle CAD$ ,  $\angle BAD$ .

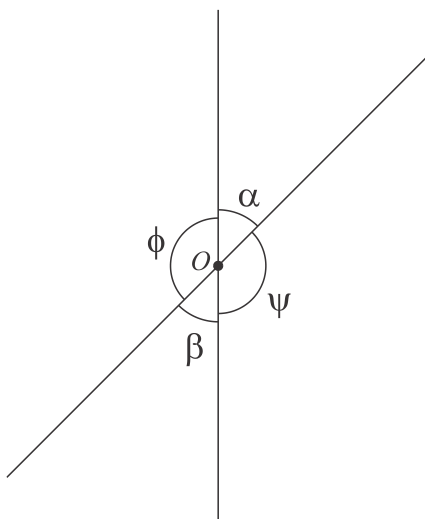
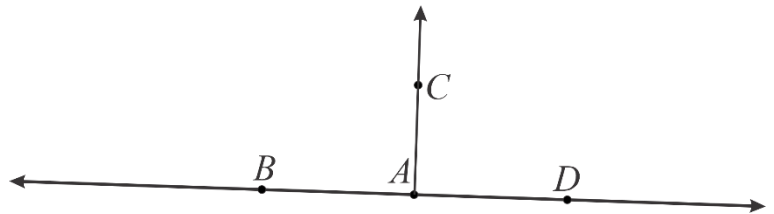


- What can you say about these angles?

Such angles we call supplementary angles.

There is only one angle which supplement itself to a straight angle. In this case supplementary angles are equal, and we call this angle a right angle.

Measure of the straight angle is  $180^\circ$ ,  
measure of the right angle is  $90^\circ$



When two straight lines intersect at a point, four angles are formed. A pair of angles opposite each other formed by two intersecting straight lines that form an "X"-like shape, are called vertical angles, or opposite angles, or vertically opposite angles.

$\alpha$  and  $\beta$  and  $\phi$  and  $\psi$  are 2 pairs of vertical angles.

**Vertical angles theorem:**

Vertical angles are equal.

In mathematics, a **theorem** is a statement that has been proven on the basis of previously established statements.

According to a historical legend, when Thales visited Egypt, he observed that whenever the Egyptians drew two intersecting lines, they would measure the vertical angles to make sure that they were equal. Thales concluded that one could prove that vertical angles are always equal and there is no need to measure them every time.

An **axiom** is a statement that is taken to be true, to serve as a starting point for further reasoning and arguments. The word comes from the Ancient Greek word ἀξίωμα (*axíōma*), meaning 'that which is thought worthy or fit' or 'that which commends itself as evident'.

**Axiom** (axiom of the straight line). Through any two points, one and only one straight line can be drawn.

**Axiom of the straight line:** Through any two points, one and only one straight line can be drawn

In mathematics a **theorem** is a statement that has been proven, or can be proven.

**Proof:**

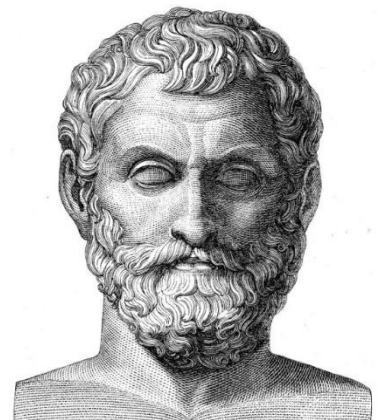
$\angle\phi + \angle\alpha = 180^\circ$  because they are supplementary by construction.

$\angle\phi + \angle\beta = 180^\circ$  because they are supplementary also by construction.

$\Rightarrow \angle\alpha = \angle\beta$ , therefore, we proved that if 2 angles are vertical angles then they are equal.

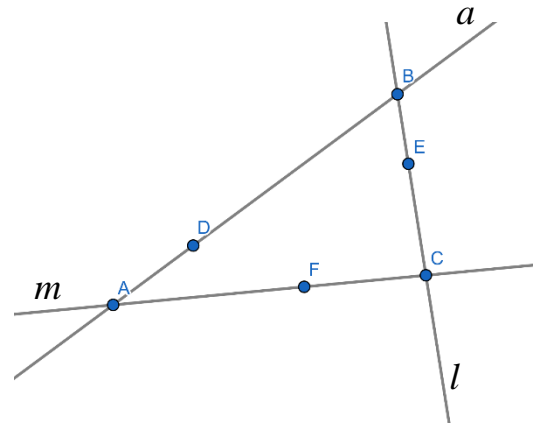
Can we say that invers is also the truth? Can we tell that if 2 angles are equal than they are vertical angels?

(**Thales of Miletus** 624-546 BC was a Greek philosopher and mathematician from Miletus. Thales attempted to explain natural phenomena without reference to mythology. Thales used geometry to calculate the heights of pyramids and the distance of ships from the shore. He is the first known individual to use deductive reasoning applied to geometry, he also has been credited with the discovery of five theorems. He is the first known individual to whom a mathematical discovery has been attributed (Thales theorem).



**Exercises:**

- Through which points does the line  $m$  pass?  
Through which points does the line  $a$  pass?  
What is the intersection of the lines  $m$  and  $l$ ?



- Mark 2 points. How many different lines can be drawn through these two points?

- Mark three points. How many lines can be drawn through three points?  
Consider all possible solution.

- Mark four points. (Any three points do not belong to the same line).  
How many lines can be drawn through four points? Through 5 points?  
10 points? 100 points?

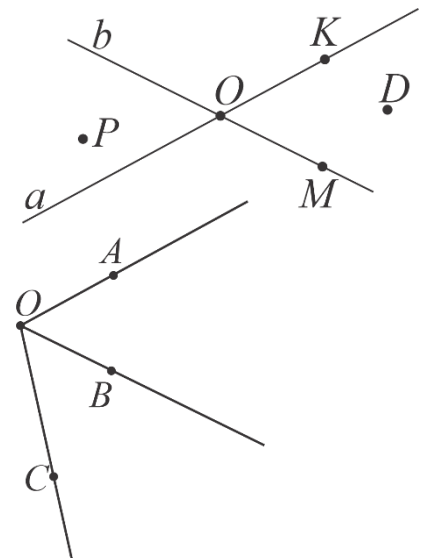
- How four points can be positioned on a plane? 5 points?

- How many intersections two non-parallel lines produce? Three non-parallel produce? 5 lines? 10 lines? 100 lines?

- Draw a line. Mark 2 points. How many segments are now on the line? Mark one more point? If four points marked on the line, how many segments they produce? 5 points? 10 points? 100 points?

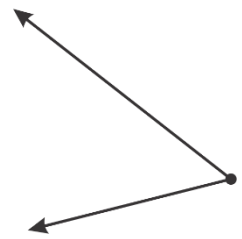
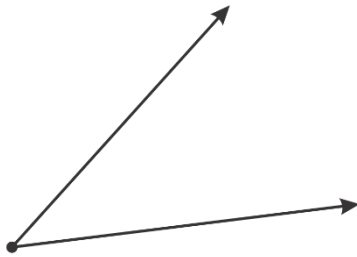
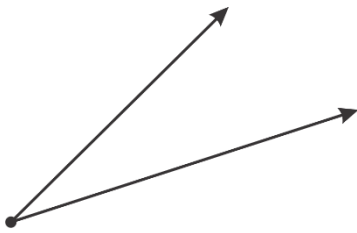
- There is a line  $a$ . Construct points A, B, and C such that the lines AB and  $a$  intersect at point C, which lies between points A and B.

- Look at the picture and answer the questions:
  - Which points belong to the lines a and b?
  - Which points do not belong to these lines?
  - Which points belong to both line a and line b



- How many angles are on the picture?

11. Measure angles:



12. Draw an angle with vertex A. From point A inside the angle, draw:

- a. two rays;
- b. three rays.
- c. 4 rays
- d. 5 rays
- e. 10 rays

How many angles do you see now in each drawing?

13. 4 angles are formed at the intersection of 2 lines. One of them is  $30^\circ$ . What is the measure of 3 others?

14. Do the operations with angular measures:

a.  $25^\circ 36' 24'' + 36^\circ 24' 40''$

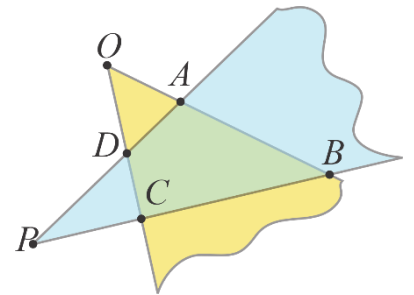
b.  $48^\circ 26' + 28^\circ 36' 34''$

c.  $48^\circ 48' 48'' - 24^\circ 36' 36''$

d.  $3 \cdot 24^\circ 36'$

15. Quadrilateral ABCD is in intersection of the angles  $\angle DOA$  and  $\angle DRC$ . Draw two angles so that they intersect by

- a. a point;
- b. a ray;
- c. a triangle;
- d. an angle;
- e. a segment.



16. Which angle is an intersection of the angles  $\angle MOL$  and  $\angle POH$  and which angle is a union of these angles?

