Math 5b. Classwork 10.

Rational numbers.

Now consider $2.4\overline{0}$ and $2.3\overline{9}$

$x = 2.4\overline{0}$
$10x = 24.\overline{0}$
$100x = 240.\overline{0}$

$100x - 10x = 240 - 24$
$x = \frac{240-24}{90} = \frac{216}{90} = 2.4$

$x = 2.3\overline{9}$
$10x = 23.\overline{9}$
$100x = 239.\overline{9}$

$100x - 10x = 239 - 23$
$x = \frac{239-23}{90} = \frac{216}{90} = 2.4$

More problems of representation of the infinite periodical decimals as fractions.

1. Represent as fractions;
   $1.23\overline{56}$, $5.4\overline{345}$

2. Represent as decimal:
   $\frac{2}{5}$; $\frac{2}{9}$; $\frac{2}{8}$; $\frac{2}{15}$; $\frac{3}{16}$;

3. Evaluate:

$$\frac{\left(\left(2.375 + \frac{15}{46}\cdot\frac{23}{30} + 0.625\right) : 4\frac{7}{8} + 2.5 : 1.25 : 3\right)\cdot 0.75}{9.56\cdot\frac{3}{4} - \frac{3}{4}\cdot 5.6 + \frac{1}{25}\cdot 0.75}$$

Equations.

1. Write without parenthesis:
   $a.\ -(a-b)$; $b. -(c+d)$; $c.\ -(-x+y)$; $d. -m+(a-c)$;

   $e.\ d-(-k+m)$; $f.\ p-(-n+k-l)$; $g.\ c-(b+c-a)+(-a+b)$;

   $h. (d-m)-b-(-m+x+d)+x$; $i.\ k-(y-c)+(d-c-y)+(-p+d)$

2. Solve the equations;
   $a.\ 2a-(14-3a) = -10$; $b.\ (9-2b)-(b-5) = 16$ $c. -(4c-7) = 5c+(11-7c)$

   $d.\ 4.2 - 0.2 : \left(\frac{1}{6} + 3b\right) = 3\frac{3}{5}$

Algebraic expression.

Expressions where variables, and/or numbers are added, subtracted, multiplied, and divided.

For example:

$$2a; \quad 3b + 2; \quad 3c^2 - 4xy^2$$

We can do a lot with algebraic expressions, even so we don't know exact values of variables. First, we always can combine like terms:

$$2x + 2y - 5 + 2x + 5y + 6 = 2x + 2x + 5y + 2y + 6 - 5 = 4x + 7y + 1$$

We can multiply an algebraic expression by a number or a variable:

$$3 \cdot (1 + 3y) = 3 \cdot 1 + 3 \cdot 3y = 3 + 9y$$

In this example the distributive property was used. Using the definition of multiplication we can write:

$$3 \cdot (1 + 3y) = (1 + 3y) + (1 + 3y) + (1 + 3y) = 3 + 3 \cdot y = 3 + 9y$$

Another example:

$$5a(5 - 5x) = \underbrace{(5 - 5x) + (5 - 5x) + \cdots + (5 - 5x)}_{5a\ times} = \underbrace{5 + 5 + \cdots + 5}_{5a\ times} - \underbrace{5x - 5x - \cdots - 5x}_{5a\ times}$$

$$= \underbrace{5 + 5 + \cdots + 5}_{5a\ times} - \underbrace{5x - 5x - \cdots - 5x} = 5a \cdot 5 - 5a \cdot 5x = 25a - 25ax$$

$$5a\ times$$

If we need to multiply two expressions

$$(a + 2) \cdot (a + 3)$$

We can use a substitution technic, we will substitute one of the expressions with a variable, for example, instead of $(a + 2)$ we can use u.

$$(a + 2) = u$$

And then we will multiply

$$u \cdot (a + 3) = u \cdot a + 3u$$

We know, that actually u should not be there, $(a + 2)$ should.

$$u \cdot (a + 3) = (a + 2) \cdot a + 3(a + 2)$$

We know how to multiply an expression by a variable (or number):

$(a + 2) \cdot a + 3(a + 2) = a \cdot a + 2a + 3a + 3 \cdot 2 = a^2 + 5a + 6$

$$(a + 2) \cdot (a + 3) = a \cdot a + 3a + 2a + 3 \cdot 2 = a^2 + 5a + 6$$

$$(a + 2) \cdot (a + 3) = a^2 + 5a + 6$$

There are a few very useful products:

$$(a+b)^2=(a+b)\cdot(a+b)=a\cdot a+a\cdot b+b\cdot a+b\cdot b=a^2+2ab+b^2$$

Let's do a few examples:

$$(2+x)^2=(2+x)(2+x)=2\cdot 2+2\cdot x+x\cdot 2+x\cdot x=2^2+2x+2x+x^2=x^2+2\cdot 2x+4=x^2+4x+4$$

$$(ab+2y)^2=(ab+2y)(ab+2y)=ab\cdot ab+ab\cdot 2y+2y\cdot ab+2y\cdot 2y=a^2b^2+2yab+2yab+4y^2$$
$$=a^2b^2+4yab+4y^2$$

$$(a-b)(a+b)=a\cdot a+a\cdot b-a\cdot b+b\cdot b=a^2-b^2$$

Exercises.

1. Multiply.

$a.$ $(a+2)(a+2)$; $f.$ $(a+1)(a+3)$; $b.$ $(3+y)(y+4)$; $g.$ $(c+d)(c-2d)$;

$c.$ $(3+x)(3-x)$; $h.$ $(y-2)(3-y)$; $d.$ $(x-y)(x+y)$; $i.$ $(x-m)(x-m)$;

$e.$ $(2a+c)(a+ac)$; $j.$ $(2d+3l)(2d+3l)$

2. Solve the equations:

$a.$ $3-\left(\frac{2}{9}m+\frac{1}{6}\right)$; $b.$ $2.6y-0.2(3y-9)=-0.5\cdot(2y+6)$; $c.$ $\frac{5}{12}\cdot(c-3)-\frac{1}{6}(2c-7)=2$