Math 5b. Classwork 7.

Exercises:

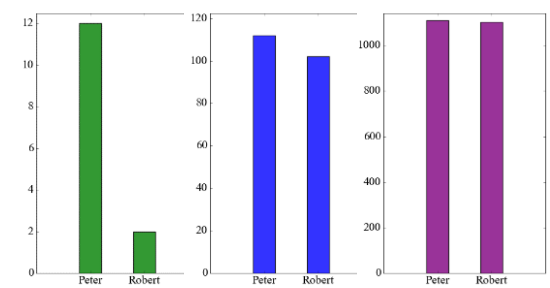
1. In which number system is the equation 2 · 3 = 10 true? 2 · 3 = 11?
2. Write in base 3 system the number 111.
3. Write in decimal system the number 111_3.
4. Do the addition $123_4 + 321_4$. Check it by transferring the numbers into decimal, adding them and transfer the answer into the 4-base system.

Ratio and proportions.

Peter has 10 dollars more than Robert. Is this a big difference? How we can compare the amount of money they have?
Take a look at the table

Peter	\$12	\$112	\$1112
Robert	\$2	\$102	\$1102

In all these cases the absolute difference is the same, but in the first case Peter has 6 times as much as Robert, in the last situation they both have almost the same amount of money. The ratios of the amount of Peter's money and Robert's money are.



$$\frac{12}{2}; \qquad \frac{112}{102}; \qquad \frac{1112}{1102};$$

The amount of money Peter and Robert have in the first case is 12 and 2 dollars and the ratio is $\frac{12}{2} = 6$, or 6:1, or 6 to 1.

Example1: (it's not a real recipe) The ratio of water and lemon juice in lemonade is 4 to 1. What does it mean? In means that for each part of lemon juice we need to add 4 parts of water, or the volume of lemon juice and water should have the same ration as 1 to 4:

$$\frac{volume\ of\ juice}{volume\ of\ water} = \frac{1}{4}$$

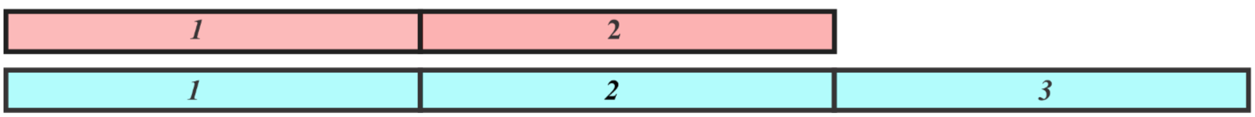
How much of lemon juice and water we need to prepare 1 l. of lemonade? 1.5 l.?

Total number of volume "units" is $5 = 4 + 1$, for total volume, $\frac{1}{5}$ is juice, and $\frac{4}{5}$ is water.
We want to have sweet lemonade and we add sugar. The ratio of water, lemon juice and sugar is 4:1:0.5 (or it can be rephrased as 8:2:1). For each part of sugar, we will use 2 parts of lemon juice, and 8 parts of water.

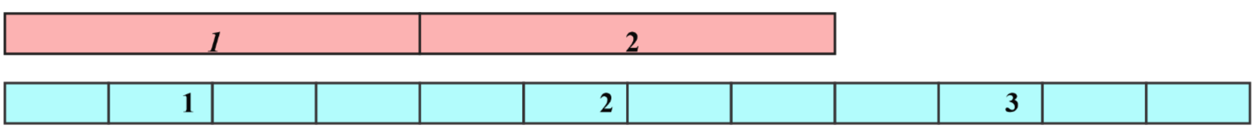
We can write the ratio of two numbers in the several ways:

$$a \text{ to } b, \qquad a\!: b, \qquad \frac{a}{b}$$

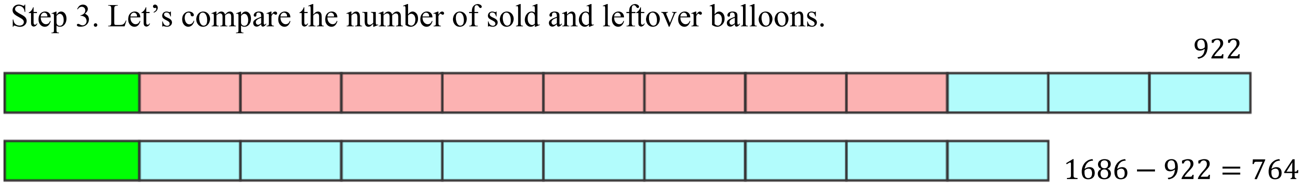
Problem:
Irene has a total of 1686 red, blue and green balloons for sale. The ratio of the number of red balloons to the number of blue balloons was 2:3. After Irene sold 3/4 of the blue balloons, 1/2 of the green balloons and none of the red balloons, she has 922 balloons left. How many blue balloons did Irene have at first?
Step 1. For each 2 red balloons there are three blue balloons, so we can show all red and blue balloons as:



We took as "unit" a half of the red balloons. The number of blue balloons is $\frac{3}{2}$ times more than number of red balloons (or three times as much as a half of the red ones)



Step 2. $\frac{3}{4}$ of the blue balloons were sold. We can't divide 3 "units" into 4 parts, without getting fractions. So, let's find LCM of 3 and 4 and divide the number of blue balloons into 12 parts.
Step 3. Let's compare the number of sold and leftover balloons.



Number of sold and unsold green balloons are the same, red balloons are all left, as well as $\frac{1}{4}$ of blue balloons. As we can see 2 small "units" of blue balloons are $922 - 764 = 158$, or one such "unit" is 79. Total amount of blue balloons is $158 \cdot 6 = 948$. The number of red balloons is

$$\frac{2}{3} \cdot 948 = 632.$$

Number of green ones is $1686 - (632 + 948) = 106$. Can we solve the problem by writing equations?

Let's try.

$G + B + R = 1686$

$3R = 2B$

$\frac{1}{2}G + R + \frac{1}{4}B = 922$

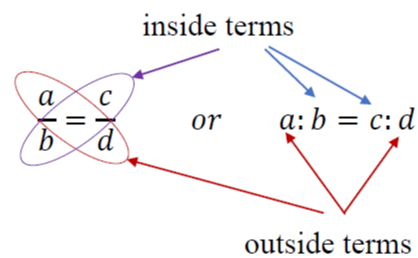
$\frac{1}{2}G + R + \frac{1}{4}B - \left(\frac{1}{2}G + \frac{3}{4}B\right) = 922 - 764$

$R - \frac{1}{2}B = 158$

$\frac{2}{3}B - \frac{1}{2}B = 158 \quad \Rightarrow \quad \left(\frac{4}{6} - \frac{3}{6}\right)B = 79 \quad \Rightarrow \quad B = 6 \cdot 158$

To cook a raspberry jam according to recipe I need to combine three cups of berries and 2 cups of sugar, or for each 3 cups of raspberries go 2 cups of sugar; ratio of raspberries and sugar (in volume) is $3{:}\,2$. If I bought 27 cups of raspberries, how many cups of sugar do I need to put to my jam?

$$\frac{3}{2} = \frac{27}{x}$$



Two ratios which are equal form a proportion.
Proportions have several interesting features.

1. The products of inside and outside terms are equal.

$$\frac{a}{b} = \frac{c}{d} \quad \Leftrightarrow \quad a \cdot d = b \cdot c$$

It can be easily shown:

$$\frac{a}{b} = \frac{c}{d} \Leftrightarrow \frac{adb}{b} = \frac{cdb}{d} \Leftrightarrow ad = cb$$

2. Also, two inverse ratios are equal:

$$\frac{a}{b} = \frac{c}{d} \quad \Leftrightarrow \quad \frac{b}{a} = \frac{d}{c}$$

Indeed:

$$\frac{a}{b} = \frac{c}{d} \quad \Leftrightarrow \quad a \cdot d = b \cdot c \quad \Leftrightarrow \quad \frac{ad}{ac} = \frac{bc}{ac} \quad \Leftrightarrow \quad \frac{d}{c} = \frac{b}{a}$$

3. Two outside terms can be switched:

$$\frac{a}{b} = \frac{c}{d} \quad \Leftrightarrow \quad \frac{d}{b} = \frac{c}{a}$$

$$\frac{a}{b} = \frac{c}{d} \quad \Leftrightarrow \quad a \cdot d = b \cdot c \quad \Leftrightarrow \quad \frac{ad}{ab} = \frac{bc}{ab} \quad \Leftrightarrow \quad \frac{d}{c} = \frac{b}{a}$$

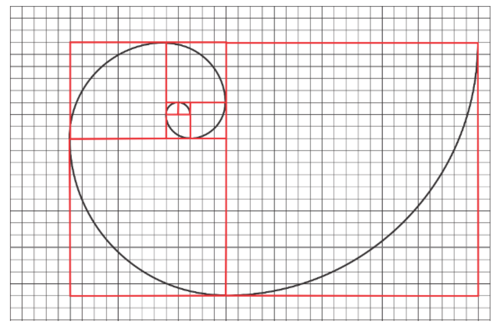
4. Two inside terms can be switched as well.

$$\frac{a}{b} = \frac{c}{d} \quad \Leftrightarrow \quad \frac{a}{c} = \frac{b}{d}$$

Famous ratios.

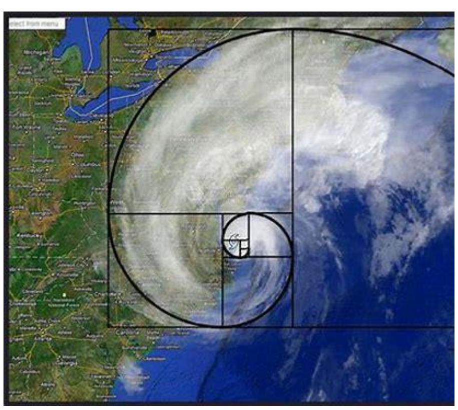
Let's measure the circumference and the diameter of a circle.

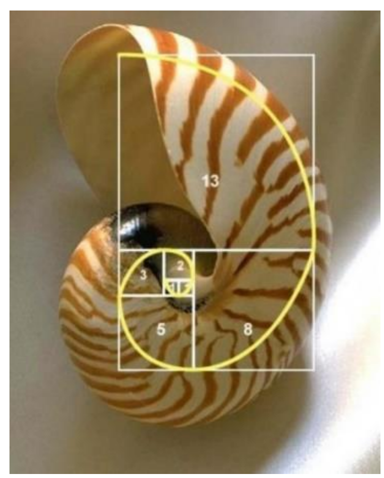
$$\frac{l}{d} = \pi$$



Fibonacci sequence:

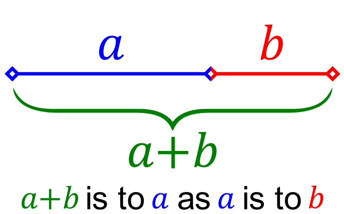
$$1, 1, 2, 3, 5, 8 \ldots ..$$
$$F_n = F_{n-1} + F_{n-2}$$

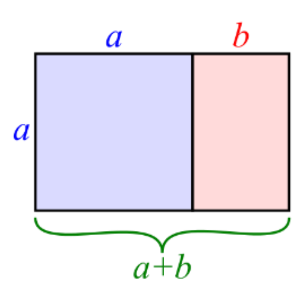




Golden ratio

$$\frac{a+b}{a} = \frac{a}{b} \cong 1.618$$





Ratio of two consecutive Fibonacci numbers becoming closer and closer to golden ratio when numbers increase.

Terms of Fibonacci Sequence	
F0 = 0	F10 = 45
F1 = 1	F11 = 89
F2 = 1	F12 = 134
F3 = 2	F13 = 233
F4 = 3	F14 = 377
F5 = 5	F15 = 610
F6 = 8	F16 = 987
F7 = 13	F17 = 1597
F8 = 21	F18 = 2584
F9 = 34	F19 = 4181

$\frac{8}{5} = 1.6;$ $\frac{13}{8} = 1.625;$ $\frac{21}{13} = 1.615 \ldots;$ $\frac{34}{21} = 1.619 \ldots$

Exercises (cont):

5. Five classes of mathematics a day, is it a lot? Five classes of mathematics a year, is it a lot?

6. If we increase the weight of an ant by 1g, will it be a significant increase (average weight of an ant is less than 10mg)? If we increase the weight of an elephant by 1 g?

   Come up with your own examples when increasing the value by the same amount can give a completely different qualitative result.

7. In a dried fruit mix, there are 7parts of dried apples, 4 parts of dried pears and 5 parts of dried apricots. (So, it can be said, that the amount of apples, pears, and apricots should be mixed in the ratio 7:4:5). What is the weight (how many grams) of apples, pears, and apricots in the fruit mix, if the total weight of the mix is 1600g?

8. In order to prepare a homemade dried fruits and nuts mix Mary took 6 parts of raisins, 5 parts of dried cranberries and 3 parts of walnuts. Cranberries and walnuts altogether weighted 2 kg 400 g. What was the weight of the mix that Mary prepared?

9. Mr. Robinson was paid $590 for a job that required 40 hours of work. At this rate, how much should he be paid for a job requiring 60 hours of work?

10. If two pounds of meat will serve 5 people, how many pounds will be needed to serve 13 people?

11. 6 oxen or 8 cows can graze a field in 28 days. How long would 9 oxen and 2 cows take to graze the same field?