Math 5b. Classwork 5.

Numeral systems.

Can non-decimal place-value system be created? For example, with base 5?
Let see, how we can create this kind of system (we use our normal digits).

Num_{10}	1	2	3	4	5	6	7	8	9	10
Num_5	1	2	3	4	10	11	12	13	14	20

11	12	13	14	15	16	17	18	19	20
21	22	23	24	30	31	32	33	34	40

21	22	23	24	25	26	27	28	29	30
41	42	43	44	100	101	102	103	104	105

We only have 5 digits (0, 1, 2, 3, 4), and 4 first "natural" numbers in such system will be represented as one-digit numbers. Number 5 then should be shown as a 2-digit number, with fist digit 1 (place - value equal to 5^1) and 0 of "units". Any number is now written in the form

$$\dots + 5^3 \cdot n + 5^2 \cdot m + 5^1 \cdot k + 5^0 \cdot p, \qquad n, m, k, p \quad \text{are} \quad 0, 1, 2, 3, 4$$

$33 = 25 + 5 + 3 = 5^2 \cdot 1 + 5^1 \cdot 1 + 5^0 \cdot 3 = 113_5$

$195 = 125 + 25 \cdot 2 + 5 \cdot 4 = 5^3 \cdot 1 + 5^2 \cdot 2 + 5^1 \cdot 4 + 0 = 1240_5$

And vice versa, we can transform the number form 5-base to decimal system:

$2312_5 = 5^3 \cdot 2 + 5^2 \cdot 3 + 5^1 \cdot 1 + 2 = 250 + 75 + 5 + 2 = 332$

Let's do the addition of 1240_5 and 2312_5 (I omitted $_5$ notation):

$$\begin{array}{r} 1240 \\ +\ \underline{2312} \\ 4102 \end{array}$$

$4102_5 = 5^3 \cdot 4 + 5^2 \cdot 1 + 5^1 \cdot 0 + 2 = 125 \cdot 4 + 25 \cdot 1 + 0 + 2 = 500 + 25 + 2 = 527$

Let's try to introduce a new digit S for 10 and then create an 11 based system.

$11^2 = 121, \qquad 11^3 = 1331$

$890 = 121 \cdot 7 + 11 \cdot 3 + 10 = 11^2 \cdot 7 + 11^1 \cdot 3 + 10 = 73S_{11}$

$4S2_{11} = 11^2 \cdot 4 + 11^1 \cdot 10 + 2 = 484 + 110 + 2 = 596$

There is another very important place-value system: binary system, base 2 system where only two digits exist; 0, and 1.

Num_{10}	1	2	3	4	5	6	7	8	9	10
Num_2	1	10	11	100	101	110	111	1000	1001	1010

In this system place value of a digit is a power of 2:

$$... + 2^3 \cdot (0,1) + 2^2 \cdot (0,1) + 2^1 \cdot (0,1) + 2^0 \cdot (0,1)$$

$11 = 8 + 2 + 1 = 2^3 \cdot 1 + 2^2 \cdot 0 + 2^1 \cdot 1 + 2^0 \cdot 1 = 1011_2$

$75 = 64 + 8 + 2 + 1 = 2^6 \cdot 1 + 2^5 \cdot 0 + 2^4 \cdot 0 + 2^3 \cdot 1 + 2^2 \cdot 0 + 2^1 \cdot 1 + 2^0 \cdot 1 = 1001011_2$

$$\begin{aligned} 189 &= 128 + 32 + 16 + 8 + 4 + 1 \\ &= 2^7 \cdot 1 + 2^6 \cdot 0 + 2^5 \cdot 1 + 2^4 \cdot 1 + 2^3 \cdot 1 + 2^2 \cdot 1 + 2^1 \cdot 0 + 2^0 \cdot 1 \\ &= 10111101_2 \end{aligned}$$

Binary numbers can be transferred to decimals too. We will do it from right to left for convenience.

$$\begin{aligned} 11010111_2 &= 2^0 \cdot 1 + 2^1 \cdot 1 + 2^2 \cdot 1 + 2^3 \cdot 0 + 2^4 \cdot 1 + 2^5 \cdot 0 + 2^6 \cdot 1 + 2^7 \cdot 1 \\ &= 128 + 64 + 16 + 4 + 2 + 1 = 215 \end{aligned}$$

Exercises:

1. Add the same digit to the number 10 on the right and left so that the resulting four-digit number is divisible by 12.
2. Add one digit to the left and one digit to the right of the number 15 so that the resulting number is divisible by 15.
3. A section of the book fell out. The first page of the fallen section is numbered 387, and the number of the last page consists of the same digits, but in a different order. How many sheets fell out of the book?

4. Write a 4-digit number, each digit 1 more than the previous digit (like 2345). Then write the 4-digit number with the same digits but in opposite order (like 5432). Subtract the smaller number from the greater one. Do it two more times using different digits. What did you notice? Can you explain it?

5. Write numbers 51 and 175 in binary system.

6. Write the numbers, written in the binary system in decimal system:

   $a.\ 11011011_2;$ $b.\ 10001101_2,$ $c.\ 111111111_2$

7. Write the numbers 245 and 824 in 6-based place-value system. Remember, that in this system you will have only 0, 1, 2, 3. 4, and 5 as digits.

8. Write the numbers 234_6 and 403_6 written in the 6-based place-value system (small number 6 shows that the number is not in decimal, but in 6-based system) in decimal system.

9. Can you find out which numbers are multiplied?

$$\begin{array}{r} * * \\ \times\ \underline{* *} \\ * * \\ +\ \underline{* * *\ \ } \\ 9 * * * \end{array}$$

10. Write number 432 in 8-based (octal) system.

11. How to arrange 127 1-dollar bills in seven wallets so that any amount from 1 to 127 dollars could be issued without opening the wallets?

12. Robert thought of a number not less than 1 and not more than 1000. Julia is allowed to ask only such questions to which Robert can answer “yes” or “no” (Robert always tells the truth). Can Julia determine the hidden number in 10 questions?

13. There is a bag of sugar, a scale and a weight of 1 g. Is it possible to measure 1 kg of sugar in 10 weights?

14. Two people simultaneously set out from A to B. The first one rode a bicycle, while the second one traveled by car at a speed five times greater than the first. Halfway to the destination, the car experienced an accident, and the motorist continued the remaining journey on foot at a speed half that of the bicyclist. Who arrived in B first? Can you say, not only who’s the fastest, but also how much faster he is.