Math 56 HW 4,

#1. a. $2^4 + 2^4 = 2 \cdot 2^4 = 2^5$

b. $2^4 \cdot 2^4 = 2^{4+4} \neq (2^4)^2 = 2^8$

c. $3^2 + 3^2 + 3^2 = 3 \cdot 3^2 = 3^{2+1} = 3^3$

d. $3^2 \cdot 3^2 \cdot 3^2 = 3^{2+2+2} = 9^{3 \cdot 2} = 3^6 = (3^2)^3 = 3^6$.

e. $\underbrace{3^4 + 3^4 + \ldots + 3^4}_{9 \text{ times}} = 9 \cdot 3^4 = 3^2 \cdot 3^4 = 3^6$

#2. a. $-4 - (-9) = -4 + 9 = 5$

b. $-(-8 + (-4)) = -(-8 - 4) = -(-12) = 12$

c. $-3 - (9 + (-6)) = -3 - (9 - 6) = -3 - (+3) = -3 - 3 = -6$

#2 d. $-3-(-7)+(-5) = -3+7-5 = -1$

e. $-2\cdot(-5)\cdot(-2) = -20$

f. $-\frac{3}{5}-\left(-1\frac{1}{3}\right) = -\frac{3}{5}-\left(-\frac{4}{3}\right) = -\frac{3}{5}+\frac{4}{3} = -\frac{9}{15}+\frac{20}{15} = \frac{11}{15}$

#3.

a. $\frac{15\cdot 9-15\cdot 6}{9\cdot 30} = \frac{15(9-6)}{9\cdot 30} = \frac{15\cdot 3}{9\cdot 30} = \frac{1}{3\cdot 2} = \frac{1}{6}.$

b. $\frac{17\cdot 4+17\cdot 9}{34\cdot 52} = \frac{17\cdot(4+9)}{34\cdot 52} = \frac{17\cdot 13}{34\cdot 52} = \frac{1}{2\cdot 4} = \frac{1}{8}$

c. $\frac{18\cdot 7+18\cdot 3}{1200} = \frac{18(7+3)}{1200} = \frac{18\cdot 10}{1200} = \frac{18}{120} =$

$= \frac{6\cdot 3}{6\cdot 20} = \frac{3}{20}.$

d. $\frac{24\cdot 11-24\cdot 3}{300} = \frac{24\cdot(11-3)}{300} = \frac{24\cdot 8}{300} = \frac{3\cdot 8\cdot 8}{3\cdot 100} = \frac{4}{25}$

#4.

a. $\frac{3}{4} + \left(\frac{5}{8} + t\right) = \frac{11}{12} + \frac{7}{8}$

$\frac{3}{4} + \frac{5}{8} + t = \frac{11}{12} + \frac{7}{8}$

$t = \frac{11}{12} + \frac{7}{8} - \frac{5}{8} - \frac{3}{4} = \frac{11}{12} + \frac{2}{8} - \frac{3}{4} = \frac{11}{12} + \frac{1}{4} - \frac{3}{4} =$

$= \frac{11}{12} - \frac{1}{2} = \frac{11}{12} - \frac{6}{12} = \frac{5}{12}$

b. $\left(\frac{4}{5} - x\right) - \frac{1}{3} = \frac{1}{6} - \frac{1}{10}$

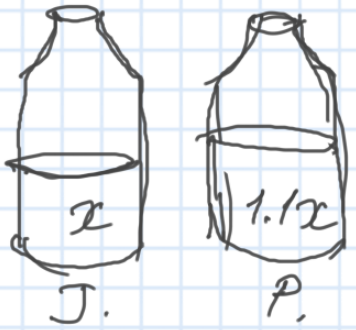
$\frac{4}{5} - x = \frac{1}{6} - \frac{1}{10} + \frac{1}{3}$

$-x = \frac{1}{6} - \frac{1}{10} + \frac{1}{3} - \frac{4}{5} = \frac{1}{6} - \frac{1}{10} + \frac{2}{6} - \frac{8}{10} = \frac{1}{2} - \frac{9}{10}$

$$-k = \frac{1}{2} - \frac{9}{10} = \frac{5}{10} - \frac{9}{10} = -\frac{4}{10}$$

$$k = \frac{4}{10} = 0.4.$$

#5.



In John's bottle there are x ml of soda.

In Peter's bottle there are $1.1 \cdot x$ ml.

(10% more means $x + 10\%$ of x or $x + 0.1x = 1.1x$)

after they drank some soda, John has $0.98x$ and Peter has $0.89 \cdot 1.1x = 0.979x$.

John has more.

$$\begin{array}{r} LOGIC \\ +\ LOGIC \\ \hline PROLOG \end{array}$$

$P = 1$.

$O = 0$.

$I = 5$

$$\begin{array}{r} LOG5C \\ +\ LOG5C \\ \hline 1ROLOG \end{array}$$

$G = 2, 3\ 4$, but has to be even, so G can be 2 or 4.

if G is 2, $L = 5$, but $I = 5$, so G is 4.

$$\begin{array}{r} L045C \\ +\ L045C \\ \hline 1ROLO4 \end{array}$$

$C = 2$

$L = 9$

$R = 8$

$$\begin{array}{r} 90452 \\ +\ 90452 \\ \hline 180904 \end{array}$$

#7. $10x + y = 5(x + y)$

x and y are one-digit numbers.

$10x + y = 5x + 5y$

$10x - 5x + y - 5y = 0.$

$5x - 4y = 0.$ It is possible only if

$x = 4$ and $y = 5.$

$5 \cdot 4 - 4 \cdot 5 = 0.$ Number is 45.

#8. $9 + 9 + 9 = 27.$ $27 \cdot 2 = 54.$

The numbers should be two-digit numers.

$10x + y = 2(x + y) = 2x + 2y$

$10x - 2x + y - 2y = 0.$

$8x - y = 0.$

$x = 1 \quad y = 8$ Number is 18.

(Another possible number is 0).

HW #3 Math 56.

#1 $2^5 = 32$ (pw).

$0.2^6 = \left(\frac{2}{10}\right)^6 = \frac{64}{1000000} = 0.000064$ (pw)

$10^7 = 10000000$ (pw)

$(-2)^5 = -32$ (pw)

$(-0.2)^6 = 0.000064$ (pw).

$(-10)^7 = -10000000$ (pw)

$(-3)^4 = 81$ (pw)

$(-0.05)^2 = 0.0025$ (pw)

$(-0.1)^3 = -0.001$ (pw).

$-2^5 = -32$ (opp. to p.w)

$-0.2^6 = -0.000064$ (opp)

$-10^7 = -10000000$ (opp)

$3^4 = 81$ (pw)

$0.05 = 0.0025$ (pw)

$0.1^3 = 0.001$ (pw)

$-3^4 = -81$ (opp. to pw)

$-0.05^2 = -0.0025$ (opp to p)

$-0.1^3 = -0.001$ (opp. to p.)

#2

$128 = 2^7$

$-128 = (-2)^7$

$0.0016 = (0.04)^2 = (0.2)^4$

$-0.0016 = -0.04^2 = -0.2^4$,

$0.0009 = 0.03^2$

$-0.0009 = -0.03^2$

#3. $\frac{1}{2} = 2^{-1}$, $\frac{1}{3} = 3^{-1}$

$\frac{1}{25} = 25^{-1} = 5^{-2}$

$\frac{1}{27} = 27^{-1} = 3^{-3}$

$\frac{1}{125} = 125^{-1} = 5^{-3}$.

A4.

a. $29^2 < 30^2$

$30^2 = 900$

$900 < 1000$

$29^2 < 30^2 = 900 < 1000$

$29^2 < 1000$

b. $48^2 < 3000$

$48^2 < 50^2$

$50^2 = 2500$

$2500 < 3000$

$48^2 < 3000$.

c. $42^2 > 1500$

$42^2 > 40^2$.

$40^2 = 1600$.

$1500 < 1600 = 40^2 < 42^2$.

$42^2 > 1500$.

d. $67^2 > 3500$

$67^2 > 60^2$

$60^2 = 3600$.

$3500 < 3600 = 60^2$

$3500 < 67^2$

#5.

a. $\left(4\frac{1}{6}\cdot 3\right):\left(7\cdot\frac{5}{21}\right)-1\frac{3}{4}\cdot 4=\frac{25\cdot 3}{6_2}\cdot\frac{21^3}{5\cdot 7}-\frac{7}{4}\cdot 4$

$=\frac{5\cdot 3}{2}-7=\frac{15}{2}-\frac{14}{2}=\frac{1}{2}.$

b. $\left(4\frac{2}{5}+3\frac{4}{5}\right)-\left(12-8\frac{1}{5}\right)=\left(4+\frac{2}{5}+3+\frac{4}{5}\right)-\left(12-8-\frac{1}{5}\right)$

$=7+\frac{6}{5}-\left(4-\frac{1}{5}\right)=7+\frac{6}{5}-4+\frac{1}{5}=3+\frac{7}{5}=4\frac{2}{5}$

#6. $\frac{3^{10}\cdot(3^2)^4}{(3^5)^3\cdot 3}=\frac{3^{10}\cdot 3^8}{3^{15}\cdot 3}=\frac{3^{18}}{3^{18}}=1.$

#7.

1. $2x > 70$ F
2. $x < 100$ T.
3. $3x > 25$
4. $x \geq 10$ F.
5. $x > 5$.

1. Let #1 be True.

$2x > 70$, T., $x > 35$

if so, then

$x > 5$ T, $3x > 25$ T, $x \geq 10$ T.

4 out of 5 statments are True.

but only 3 should be T.

$2x > 70$ is False

$2x \leq 70$.

If $2x \leq 70$, $x < 100$ is True.

If #4 is T, #5 is also T, and #3 is True as well.

we have 4 True statement, so #4 should be false. Iso, #3, #5 are both T.

Only number 9 makes it possible.

$x < 10$, $x > 5$, $3x > 25$. $x = 9$.

#8.

The difference between two bags are 2 apples and $600 - 400 = 200$ g.

So 1 apple is 100 g.

$200 + 10p = 400$

10 plums are 200 g, 1 plum is $200 : 10 = 20$ g.

or

$4a + 10p = 600$

$2a + 10p = 400$

$4a - 2a + 10p - 10p = 600 - 400$

$2a = 200$

$a = 100$

9.

1. yellow
2. green
3. pink.

HW #2.

#1.

a. $21^2 = 441$

$29^2 = 841$.

b. $34^2 = 1156$

$36^2 = 1296$

c. $75^2 = 5625$

d. $23^2 = 529$

$27^2 = 729$.

#2.

$(-3)^2 = 9$ $\quad (-3)^3 = -27$ $\quad (-2)^7 = -128$

$-3^2 = -9$ $\quad 2^7 = 128$ $\quad -2^7 = -128$

$(2 \cdot 3)^3 = 6^3$ $\quad 2 \cdot 3^3 = 2 \cdot 27 = 54$

$\left(\frac{1}{3}\right)^2 = \frac{1}{9}$ $\quad \frac{1}{3^2} = \frac{1}{9}$.

#3.

$2^1 = 2$ $2^4 = 16$ $2^7 = 128$ $2^{10} = 1024$

$2^2 = 4$ $2^5 = 32$ $2^8 = 256$ $2^{11} = 2048$

$2^3 = 8$ $2^6 = 64$ $2^9 = 512$ $2^{12} = 4096$.

power	1	2	3	4	5	6	7	8	9	10	11	12
last digit.	2	4	8	6	2	4	8	6	2	4	8	6

last digit of 2^{13} will be 2.

2^{14} 4

2^{15} 8

$2^{32} \rightarrow ?$ $32 : 4 = 8$, last digit is 6.

$2^{49} \rightarrow ?$ $49 : 4 = 12 R(1)$ last digit is 2

$2^{62} \rightarrow ?$ $62 : 4 = 15 R(2)$ last digit is 4.

2022^{23} last digit will be the same as 2^{23}, $23:4=5R(3)$, last digit is 8.

2025^{23} → last digit is 5.

2023^{23}.

$3^1=3$
$3^2=9$
$3^3=27$
$3^4=81$
$3^5=243$
$3^6=279$

power	1	2	3	4	5	6
last digit	3	9	7	1	3	9	7	1

3^{23} → last digit is 7.

$23:4=5R(3)$

2026^{23} → last digit is 6.

#4.

$$9^7 - 3^{10} = (3^2)^7 - 3^{10} = 3^{14} - 3^{10} = 3^{10} \cdot 3^4 - 3^{10} =$$
$$= 3^{10}(3^4 - 1) = 3^{10} \cdot (81 - 1) = 3^{10} \cdot 80 =$$
$$= 3^{10} \cdot 20 \cdot 4.$$

20 is a factor of this numer. Therfore the number is divisible by 20.

#5. Let's denote the ages of all family members as

m - mom
d - daughter
s - son
p - dad.

$m + d + s + p = 110$.
$m = 5d$, but also $m = p - 6$,
so $p - 6 = 5d$ and $p = 5d + 6$.

$5d + d + 2d + 5d + 6 = 110$.

$13d = 104$, $d = 104 : 13 = 8$.

Daughter is 8 y.o.

Son is $8 \cdot 2 = 16$ y.o.

mom is $5 \cdot 8 = 40$ yo.

dad is $40 + 6 = 46$ yo.

#6. $$\frac{1\frac{1}{2} \cdot 2\frac{2}{3} \cdot 0.36}{0.6 \cdot 2\frac{1}{4} \cdot 1\frac{1}{3}} = \frac{\frac{3}{2} \cdot \frac{8}{3} \cdot \frac{36}{100}}{\frac{6}{10} \cdot \frac{9}{4} \cdot \frac{4}{3}} = \frac{4 \cdot \frac{9}{25}}{\frac{2}{10} \cdot 9} =$$

$$= 4 \cdot \frac{9}{25} \cdot \frac{10}{2} \cdot \frac{1}{9} = \frac{4 \cdot 2 \cdot 5}{5 \cdot 5 \cdot 2} = \frac{4}{5} = 0.8.$$

#7. $2^x \cdot 2^{2x} = 64.$

a. $2^{x+2x} = 2^6$

$2^{3x} = 2^6$ $\quad 3x = 6,\ x = 2.$

b. $3^n \cdot 9 = 81$

$3^n = 9$

$n = 2$

c. $5^p = 1$. , $p = 0$.

#8.

last customer bought one apple, $\frac{1}{2} + \frac{1}{2}$.

fifth customer bought 2 apples out of 3
1 and $\frac{1}{2}$ (half of remaining apples) and $\frac{1}{2}$.

fourth customer bought 4 apples out of 7
$3\frac{1}{2}$ and $\frac{1}{2}$.

third bought 8 apples out of 15.

second bought 16 apples out of 33.

first bought 34 out of 67.

HW 1.

#1. Last digit is 0, so there are 2 and 5 among these prime numbers.

$2, 3, 5, 7 \quad 2 \cdot 3 \cdot 5 \cdot 7 = 3 \cdot 7 \cdot 10 = 210$

#2. Yes, it can. For example

$2 + 3 + 4 + 5 = 14$
$14 : 4 = 3 R (2).$

#3. Let's denote these 4 numbers as

$n, n+1, n+2, n+3.$

$n + n+1 + n+2 + n+3 = 4n + 6$

$4n + 6 = 4n + 4 + 2 = 4(n+1) + 2$

The sum is represented as $4 \cdot (n+1) + 2 = S$

If it's divided by 4: $S = (n+1) \cdot 4 + 2$

quotient, divisor, remainder

#4

$$\left(1.5 : \frac{1}{3} - \frac{3}{8} : 0.25\right) \cdot 3.2 - 3.2 \cdot \frac{5}{8} =$$

$$3.2\left(1.5 \cdot 3 - \frac{3}{8} \cdot 4 - \frac{5}{8}\right) = 3.2 \cdot \left(4.5 - \frac{3}{2} - \frac{5}{8}\right) =$$

$$= 3.2\left(4.5 - 1.5 - \frac{5}{8}\right) = 3.2\left(3 - \frac{5}{8}\right) =$$

$$= 3.2\left(\frac{24}{8} - \frac{5}{8}\right) = 3.2 \cdot \frac{19}{8} = \frac{32}{10} \cdot \frac{19}{8} = \frac{4 \cdot 8 \cdot 19}{10 \cdot 8} = \frac{2 \cdot 19}{5} = 7.6.$$

#5

$56 \frac{km}{h}$ →

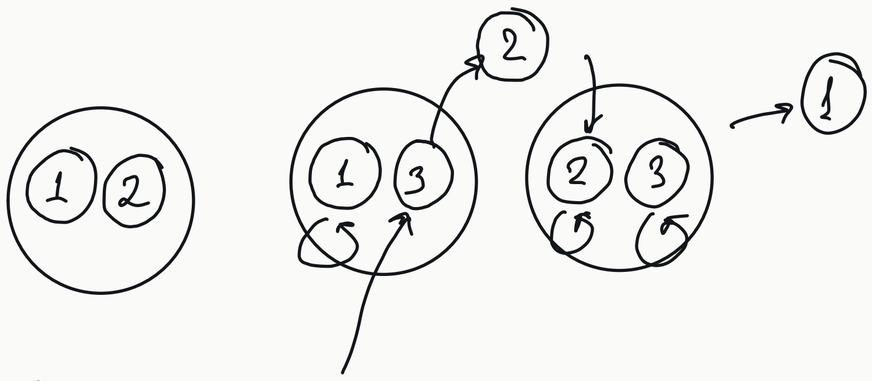
← $84 \frac{km}{h}$

1. $56 + 84 = 140 \frac{km}{h}$

2. $140 \frac{km}{h} = 140000 \frac{m}{h} = \frac{140000}{3600} \frac{m}{s}$

3. $\frac{140000}{3600} \cdot 9 = \frac{140000}{400} = 350 m.$

#6.



#7. If on 7 days el. and b.e. eat 35 basket
in 1 day they will eat 35:7 = 5 baskets.
One el. and 2 b.e. in 10 days eat 60 baskets.
So in 1 day they will eat 6 baskets.
1 baby el. eats 1 basket a day.
1 ad. eleph. eats 5-1=4 basket a day.
Also, the problem can be solved formally.
$x \rightarrow$ number of baskets el. eats per day
$y \rightarrow$ number of baskets b.e. eats per day

$(x+y)\cdot 7 = 35$ $\quad x+y=5$

$(x+2y)\cdot 10 = 60.$ $\quad x+2y=6.$

$x = 5-y$

$5-y+2y=6$

$y = 6-5=1.$

$x = 5-1=4.$

#8 a.
$$\begin{array}{r} SO \\ +\ SO \\ \hline TOO \end{array}$$
$T=1.$ $\quad O=0$ $\quad S=5$
$$\begin{array}{r} 50 \\ +\ 50 \\ \hline 100 \end{array}$$

b.
$$\begin{array}{r} US \\ +\ AS \\ \hline ALL \end{array}$$
$A=1.$ $\quad U=8$ or $9.$
if $U=9$, $L=0$ and $S=0.$
so u can't be 9
$$\begin{array}{r} 85 \\ +\ 15 \\ \hline 100 \end{array}$$
if $U=8$, $L=0$, $S=5$

c.
$$\begin{array}{r} COCA \\ +\ COLA \\ \hline OASIS \end{array}$$
$O=1.$
$$\begin{array}{r} C1CA \\ C1LA \\ \hline 1ASIS. \end{array}$$
$S=2$ or 3, but
can't be odd
so, $S=2$

$$\begin{array}{r} C1C6 \\ C1L6 \\ \hline 1A2I2 \end{array}$$
$C=5$
$A=0$
$$\begin{array}{r} 5156 \\ 51L6 \\ \hline 10212 \end{array}$$
$L=3$
$I=9.$
$$\begin{array}{r} 5156 \\ 5136 \\ \hline 10292 \end{array}$$