

Math 5b, homework 2.



1. What digits should be put instead of * to get true equality? How many solutions does each problem have?

a. $(2 *)^2 = ** 1$; b. $(3 *)^2 = *** 6$ c. $(7 *)^2 = *** 5$ d. $(2 *)^2 = ** 9$

2. Evaluate:

$$(-3)^2; \quad -3^2; \quad (-3)^3; \quad 2^7; \quad (-2)^7; \quad -2^7; \quad (2 \cdot 3)^3; \quad 2 \cdot 3^3; \quad \left(\frac{1}{3}\right)^2; \quad \frac{1}{3^2};$$

3. Let's take a look on a sequence of numbers:

$$2, \quad 2^2, \quad 2^3, \quad 2^4, \dots, \quad 2^{12}$$

What are the last digits of this numbers? Can you tell the last digit of 2^{13} , 2^{14} , 2^{15} ?

What about 2^{32} , 2^{49} , 2^{62} ?

Can you tell what would be the last digit of 2022^{23} ? 2025^{23} ? 2023^{23} ? 2026^{23}

4. Prove that

Example:

$8^5 + 2^{11}$ is divisible by 17

$$8^5 + 2^{11} = (2^3)^5 + 2^{11} = 2^{15} + 2^{11} = 2^{11} \cdot 2^4 + 2^{11} \cdot 1 = 2^{11}(2^4 + 1) = 2^{11} \cdot (16 + 1) = 2^{11} \cdot 17$$

$9^7 - 3^{10}$ is divisible by 20

5. In the family, there are 4 people: mom, dad, son, and daughter. Together, they are 110 years old. The mother is 5 times older than the daughter and 6 years younger than the father, while the daughter is 2 times younger than the son. How old is each member of the family?

6. Evaluate (the answer is 0.8 (or 4/5)):

$$\frac{1\frac{1}{2} \cdot 2\frac{2}{3} \cdot 0.36}{0.6 \cdot 2\frac{1}{4} \cdot 1\frac{1}{3}};$$

7. Solve the following equations:

a. $2^x \cdot 2^{2x} = 64$; b. $3^n \cdot 9 = 81$; c. $5^p = 1$

8. The farmer brought a basket of apples to the market. To the first customer, he sold half of all his apples and half an apple more, to the second customer - half of the remainder and half an apple more, to the third - half of the remainder and half an apple more, and so on. However, when the sixth customer came and bought half of the remaining apples and half an apple, it turned out that, like the other buyers, all his apples were whole, and the farmer sold all his apples. How many apples did he bring to the market?