MATH 5: HANDOUT 13
MORE POWERS. SCIENTIFIC NOTATION.

More Powers

Recall that for a positive integer n, we have defined

$$a^n = \underbrace{a \cdot a \cdot \dots \cdot a}_{n \text{ times}}$$

then

$$a^m a^n = a^{m+n}, \qquad a^m \div a^n = a^{m-n}$$

It turns out that there is only one way to define a^n for $n = 0$ and negative n so that these rules still work:

$$a^0 = 1 \qquad a^{-n} = \frac{1}{a^n}$$

For example, $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

Another important formula is the following:

$$(a^n)^m = a^{n \times m}.$$

It is easy to see why this formula holds:

$$(a^n)^m = \underbrace{(\underbrace{a \cdot a \cdot \dots \cdot a}_{n \text{ times}}) \times \dots \times (\underbrace{a \cdot a \cdot \dots \cdot a}_{n \text{ times}})}_{m \text{ times}} = a^{n \times m}$$

Scientific notation

Scientific notation is a convenient way to write very large numbers: instead of writing $2,000,000,000$ one can say "2 and then 9 zeros". Since writing a zero at the end is the same as multiplication by 10, we can also write the same number as

$$2 \times 10 \times \dots \times 10 \qquad \text{(9 times)}$$

or, for short 2×10^9. Thus, we can write

$$2,000,000,000 = 2 \times 10^9$$

which is much shorter.

Similarly, we can write

$$\begin{aligned} 2,310,000,000 =& 231 \times 10 \times \dots \times 10 \qquad \text{(7 times)} \\ =& 2.31 \times 10 \times \dots \times 10 \qquad \text{(9 times)} \\ =& 2.31 \times 10^9 \end{aligned}$$

Such a form (a decimal with one digit before decimal point times 10 to some power) is called the *scientific notation*.

To write a number larger than 10 in scientific notation, you should:

1. Count how many digits the whole part has. The power of 10 will be number of digits minus 1.
2. Write down the digits of the number, but now put the decimal point after the first digit.

Example:

$$3412000 = 3.412000 \times 10^6 = 3.412 \times 10^6$$

In a similar way, scientific notation is very useful for very small numbers. For example, weight of one atom of hydrogen is about 1.66×10^{-24} gram — or

$$0.00000000000000000000000166 \; gr$$

Classwork

1. Simplify:

(a) $(2z^2 \cdot 3z^3 \cdot z)^2$

(b) $\left(\dfrac{5g^4b^5}{4g^2b^3}\right)^3$

(c) $2x^2 \cdot x^3 - x^7 \div x^2$

(d) $\dfrac{(-ab)^8}{(ab)^2}$

(e) $\dfrac{18^{n+3}}{3^{2n+5} \cdot 2^{n-2}}$

(f) $\left(\dfrac{3ab^3}{15b}\right)^2 \cdot \dfrac{75c}{a^2b^6}$

2. Let $x = a^3 \cdot b^2$, $y = \dfrac{b^5}{a^2c^4}$, and $z = \dfrac{c^3}{ab}$. Express in terms of a, b, c:

(a) xyz

(b) $x^2y^3z^4$

(c) $\dfrac{xy}{z}$