Math 4. Class Work 22

Exponent

Exponentiation is a mathematical operation written as a^n

$$a^n = \underbrace{a \cdot a \cdot a \dots \cdot a}_{n\ times}$$

We have two numbers, the **base *a*** and the **exponent *n*.** The exponent indicates how many copies of the base are multiplied together.

- When ***n* is a positive** integer, we have repeated multiplication of the base n times. In that case, a^n is called the ***n*-th power of *a***, or *a* raised to the power *n*.

For example, $3^5 = 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 243.$

Here, **3** is the ***base***, **5** is the ***exponent*,** and 243 is *the fifth power of 3*, or *3 raised to the fifth power*, or *3 to the power of 5*.

Properties of exponent:

$$a^{10} \cdot a^{15} = \underbrace{a \cdot a \dots \cdot a}_{10\ times} \cdot \underbrace{a \cdot a \dots \cdot a}_{15\ times} = \underbrace{a \cdot a \cdot a \dots \cdot a}_{10+15\ times} = a^{10+15} = a^{35}$$

$$(a^{10})^{15} = \underbrace{a^{10} \cdot a^{10} \cdot \dots \cdot a^{10}}_{15\ times} = \underbrace{\underbrace{a \cdot a \cdot \dots \cdot a}_{10\ times} \cdot \dots \cdot \underbrace{a \cdot a \cdot \dots \cdot a}_{10\ times}}_{15\ times} = a^{10 \cdot 15} = a^{150}$$

$$a^{10} \cdot a = \underbrace{a \cdot a \cdot a \dots \cdot a}_{10\ times} \cdot a = \underbrace{a \cdot a \cdot a \cdot a \dots \cdot a}_{10+1\ times} = a^{10+1} = a^{10} \cdot a^1$$

$a^1 = a$ and $a^0 = 1$ for any number *a,* but 0.

$$a^{10} = a^{10} \cdot 1 = a^{10+0} = a^{10} \cdot a^0$$

Also, if there are two numbers *a* and *b*:

$$(a \cdot b)^{10} = \underbrace{(a \cdot b) \cdot \dots \cdot (a \cdot b)}_{10\ times} = \underbrace{a \cdot \dots \cdot a}_{10\ times} \cdot \underbrace{b \cdot \dots \cdot b}_{10\ times} = a^{10} \cdot b^{10}$$

1. $a^n = \underbrace{a \cdot a \cdot a \ldots \cdot a}_{n\ times}$
2. $a^n \cdot a^m = a^{n+m}$
3. $(a^n)^m = a^{n \cdot m}$
4. $a^1 = a$, for any a
5. $a^0 = 1$, for any $a \neq 0$
6. $(a \cdot b)^n = a^n \cdot b^n$

- A positive number raised to any power will result in a positive number.
- A negative number, raised to a power, represented by an even number, is positive, represented by an odd number is negative.

Problems

1. Write the addition as a multiplication
   a) $2 + 2 + 2 + 2 + 2 + 2 + 2 =$

   b) $\underbrace{\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \cdots + \frac{1}{2}}_{30\ times} =$

   c) $\underbrace{5 + 5 + 5 + 5 + \cdots\ + 5}_{17\ times} =$

   d) $\underbrace{a + a + a + \cdots a}_{n - times} =$

2. The area of a square with a side $\boldsymbol{a}$ is $A = a.a$. This can also be written as $A = a^2$ and we read it as " a to the second power" or " a squared."
   The volume of a cube with a side $\boldsymbol{a}$ is $V = a.a.a$ and a^3 and we read it as "a to the third power or "a cubed."
   When we solve problems, sometimes we need to calculate a product of the same number that repeats many times.

   - Define a number raised to a power.

3. Write the following product as a base raised to a power (exponent)
   $a)\ 2.2.2.2.2 = 2^5 \quad 2\ is\ the\ base,\ \ 5\ is\ the\ exponent$
   b) $3.5 \cdot 3.5 \cdot 3.5 =$

c) $\left(-1\frac{1}{2}\right)\cdot\left(-1\frac{1}{2}\right)\cdot\left(-1\frac{1}{2}\right)\cdot\left(-1\frac{1}{2}\right) =$

d) $a\cdot a\cdot a\cdot a\cdot a\cdot a =$

e) $\underbrace{a\cdot a\cdot a\cdot a\cdot a\cdot \;\ldots\; \cdot a}_{n-\text{times}} =$

4. Write the following product as powers.
   a) $4^1 =$

   b) $(-2)^1$

   c) $2^0 =$

   d) $1\cdot 3^5 =$

5. Write the powers as a product and calculate
   a) $4^3 =$

   b) $8^2 =$

   c) $1^7 =$

   d) $(-2)^4 =$

   e) $(-2)^5 =$

   f) $-3^3 =$

   g) $(2\cdot 3)^3$

   h) $(2\cdot 5)^2 =$

   - Define $(a\cdot b)^n = a^n\cdot b^n$

6. Find the exponent so that the equation holds.
   $a)$ $8^* = 512;$ $b)$ $2^* = 64;$ $c)$ $3^* = 81;$ $d)$ $7^* = 343$

7. In a magical lake, the number of water lilies doubles every night. On March 1st, the magician planted the first lily, and in 90 nights, the entire lake was covered with lilies. On which day was only half of the lake covered?

8. Exponents can help write numbers in their expanded form. In our base-10 place value system, the position of the digit indicates which power of 10 should be multiplied by the digit. Write the numbers in expanded form.
   a) Example: $345 = 300 + 40 + 5 = 100\cdot 3 + 10\cdot 4 + 1\cdot 5 = 10^2\cdot 3 + 10^1\cdot 4 + 10^0\cdot 5$

   b) $625 =$

   c) $8705 =$

   d) $42672 =$

   e) $25 =$

9. Use the properties of the exponents to write the product as a single power(s)

Multiplication: $a^{10} \cdot a^{15} = \underbrace{a \cdot a \ldots \cdot a}_{10\ times} \cdot \underbrace{a \cdot a \ldots \cdot a}_{15\ times} = \underbrace{a \cdot a \cdot a \ldots \cdot a}_{10+15\ times} = a^{10+15} = a^{35}$

Division $\frac{a^5}{a^2} = \frac{a \cdot a \cdot a \cdot a \cdot a}{a \cdot a} = a^{5-2} = a^3$

a) $2^2 \cdot 2^4 =$

b) $5^3 \cdot 5^2 \cdot 5 =$

c) $\frac{21 \cdot 7^2}{7^3} =$

10. Imagine that you put 1000 dollars into a bank account that will give you 5% annual interest, and you do not take any money out

a) How much money will you have in 1 year?

$$1000 + \frac{5}{100} \times 1000 = 1000 + 50 = 1050 = 1.05 \cdot 1000$$

b) Next year, you will need to multiply again by 1.05 what you got the first

$$1.05 \cdot (1.05 \cdot 1000) = 1.05^2 \cdot 1000 = 1102.5$$

c) In 100 years, how much money will you have in the bank?